Free Response Final 2011 Name

1. The table below shows selected values for a twice – differentiable function f.

x	-5	-4	-3	-2	-1	0	1
$f(x)$	3	2	0	-3	0	-5	-9

(A) Find the midpoint Reimann sum approximation for $\int_{-5}^{1} f(x)\,dx$ using 3 subintervals of equal length.

(B) Show that the average rate of change of f over the interval $-5 \le x \le -2$ is the same as the average rate of change of f over the interval $-2 \le x \le 1$.

(C) Explain why there must be at least one value c such that $-2 < c < 0$ and $f'(c) = 0$.

(D) Must there be at least one value d such that $-5 < d < 1$ and $f''(d) = 0$? Explain why or why not

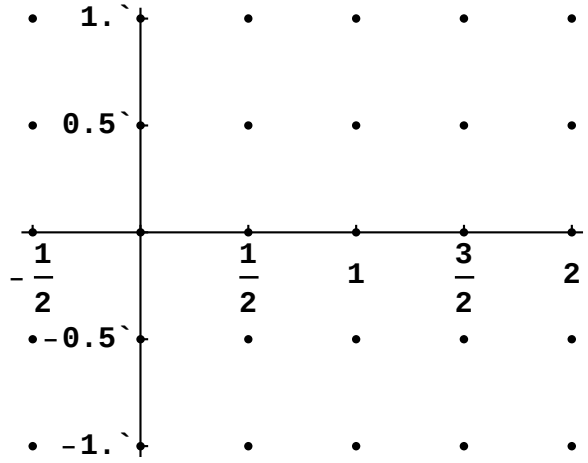
2. A function $f(x)$ satisfies the differential equation $\frac{dy}{dx} = \sqrt{1 - y^2}$ with the initial condition $y(1) = 0$

(A) Find $\frac{d^2y}{dx^2}$

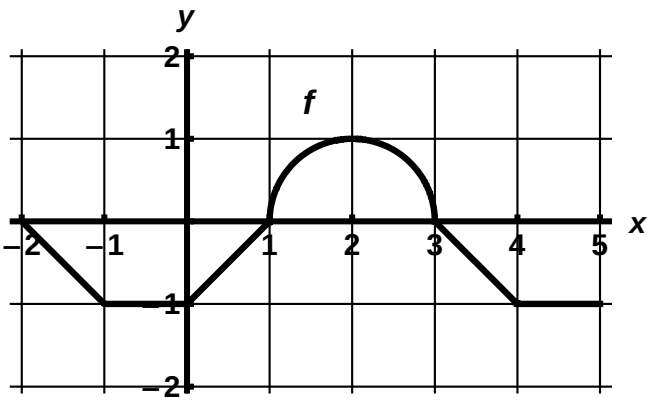
(B) Using the initial point $y(1) = 0$, and $\Delta x = \frac{1}{2}$, take two steps using Euler's Method for $\frac{dy}{dx} = \sqrt{1 - y^2}$ (to approximate the solution at $x = 2$).

(C) Solve the differential equation $\frac{dy}{dx} = \sqrt{1 - y^2}$, where $y(1) = 0$, and express your solution with y in terms of x (isolate y).

(D) Now, create a slopefield for the same differential equation in (C) for the given graph. Overlay your solution from both (B) and (C) on this slopefield



3. The graph of a function f, defined on the closed interval $-2 \le x \le 5$, is shown below. Now, let $h(x) = \int_0^{2x} f(t)\,dt$

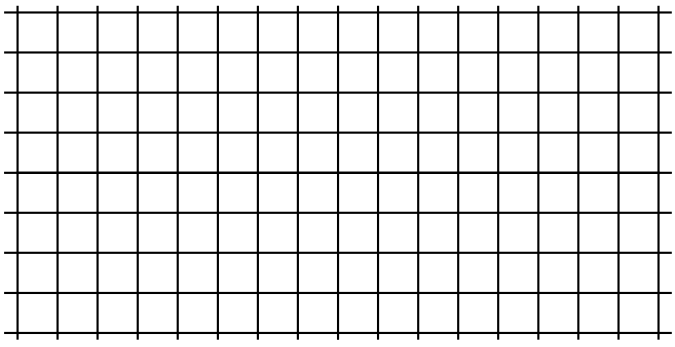


(A) Find $h(2)$ and $h'(2)$

(B) Find the x – coordinates of all local maximum points and all local minimum points of $h(x)$ over the interval $-1 \le x \le 2.5$. Justify your answer.

(C) Find the intervals where $h(x)$ is concave up. Justify your answer.

(D) Draw a sketch of both $h'(x)$ and $h(x)$. Clearly label both functions and both of the axes



4. A particle moves on the $y-$axis so that its position at any time $t \geq 0$ is given by $y(t) = t^2 - 4\ln(t+1) - 1$

(A) Find the velocity $v(t)$ at any time $t \geq 0$.

(B) For any time $t \geq 0$, find the lowest y value attained by the particle (recall that the motion is along the $y-$axis) Justify your answer.

(C) Find all values of t for which the speed of the particle is increasing. Justify your answer.

(D) Find the total distance traveled by the particle from $t = 0$ to time $t = 2$.