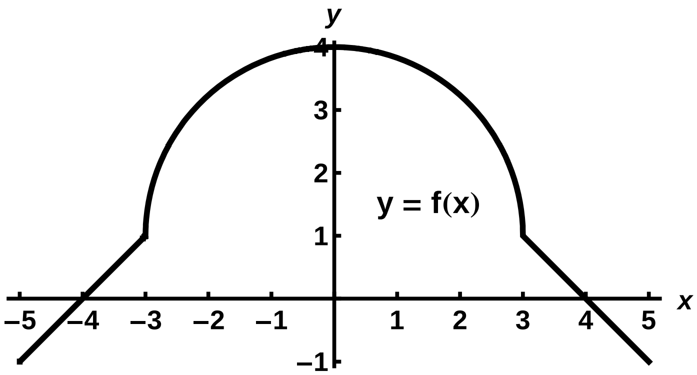
Free Response Final 2012 **Name**



1. The graph of a function $f(x)$ consists of a semicircle and two line segments as shown in the figure above. The function $g(x)$ is defined by $g(x) = \int_{-3}^{x} f(t)\, dt$

(A) Find $g(-4)$, $g(0)$, and $g(3)$

(B) Write an equation for the tangent line to the graph of $g(x)$ at $x = 3$

(C) Find all values of x, on the interval $(-5, 5)$, where $g(x)$ has relative extreme values. Be sure to label these as max or min.

(D) Find where $g(x)$ is concave up and concave down on the interval $(-5, 5)$.

2. Let f be a differentiable function whose graph passes through the point $\left(2, \frac{1}{2}\right)$. For all points (x, y), the slope on the graph of $y = f(x)$ is given by $\frac{dy}{dx} = y^3(3 - 2x)$

(A) Find $\frac{d^2y}{dx^2}$ Is the graph of f concave up, concave down, or neither at the point $\left(2, \frac{1}{2}\right)$?

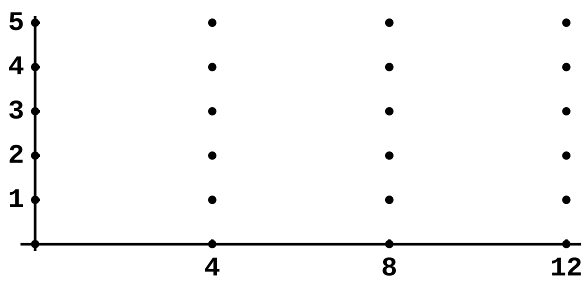
(B) Write an equation of the tangent line to the graph of f at the point $\left(2, \frac{1}{2}\right)$.

(C) Find $y = f(x)$ by solving the separable differential equation $\frac{dy}{dx} = y^3(3 - 2x)$ with the initial condition $\left(2, \frac{1}{2}\right)$

Isolate y in your solution.

3. Consider the differential equation $\frac{dy}{dt} = \frac{1}{10} y(5 - y)$

(A) On the axes provided below, sketch a slope field for the differential equation at the points indicated. Use the slope field to sketch a solution curve that passes through the point $(0, 1)$.



(B) Using the differential equation above, $\frac{dy}{dt} = \frac{1}{10} y(5 - y)$, and Euler's Method, approximate the solution at $y(8)$. Use $y(0) = 1$ and $\Delta x = 4$ (2 steps). What can you say about this approximation?

(C) The differential equation $\frac{dy}{dt} = \frac{1}{10} y(5 - y)$ should look familiar to you. What is this model called? Attempt to provide a solution to the differential equation from what you remember (just try it!)

(D) Extra Credit. Solve the differential equation $\frac{dy}{dt} = \frac{1}{10} y(5 - y)$ by the method of separation of variables.

x	-3	-2	-1	0	1
$f(x)$	7	3	1	3	7
$f'(x)$	-5	-3	0	3	5

4. Let $f(x)$ be a twice – differentiable function on the closed interval $[-3, 1]$. The table above gives the values of f and its derivative f' for selected points along the interval. The second derivative of f has the property that $f''(x) > 0$ for all x in the open interval $(-3, 1)$

(A) Evaluate $\displaystyle\int_{-1}^{0} (2f'(x) + 3f''(x))\,dx$ Show your work.

(B) Write an equation for the line tangent to the graph of f at the point $(-2, 3)$. Use this line to approximate $f(-2.1)$. Is this approximation greater than or less than the actual value of $f(-2.1)$? Justify your answer.

(C) Find a positive number b such that there must be a value c on the open interval $(0, 1)$ where $f''(c) = b$

(D) Is it possible that $f''(x)$ is increasing for all x on the open interval $(-1, 1)$? Justify your answer.