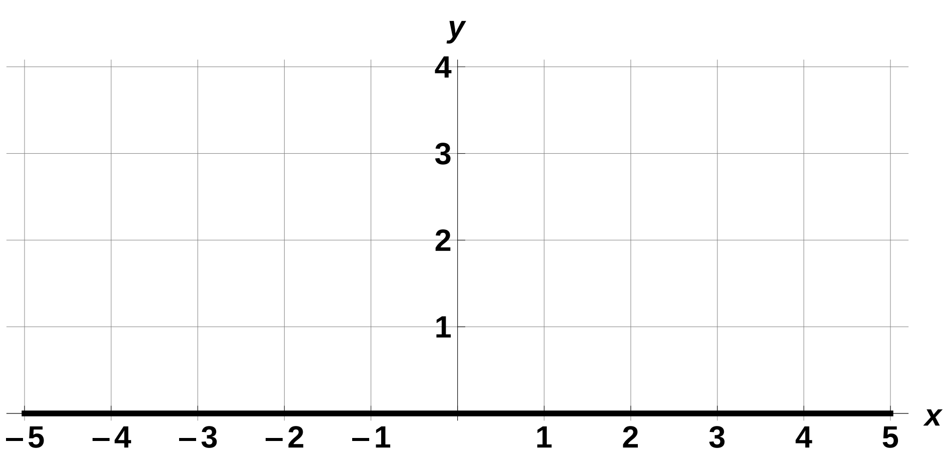
Free Response Final 2013 Name

1. Consider $f(x) = \dfrac{3x^2 + 1}{x^2 + 1}$

(A) Find $f'(x)$, and use the first derivative test to find all local extrema (x – values only).

(B) Find $f''(x)$, and find all points of inflection (x – values only).

(C) Use the information found in (A) and (B) above to draw a detailed graph of $f(x)$ below.



2.

x	2	3	5	8	13
$f(x)$	1	4	-2	3	6

Let f be a function that is twice differentiable for all real numbers. The table above gives the output values of f for selected input values on the closed interval $2 \le x \le 13$.

(A) Estimate $f'(4)$. Show the work that leads to your answer.

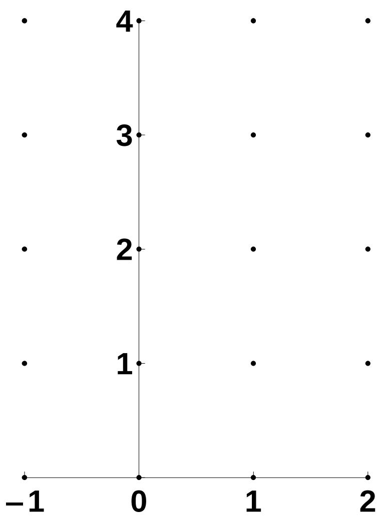
(B) Evaluate $\int_{2}^{13} (3 - 5f'(x))\,dx$. Show the work that leads to your answer.

(C) Use a left Riemann sum with subintervals indicated by the data in the table to approximate $\int_{2}^{13} f(x)\,dx$, and show the work that leads to your answer.

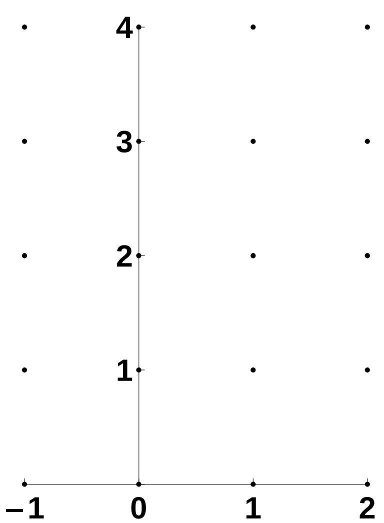
(D) Suppose $f'(5) = 3$ and $f''(x) < 0$ for all x in the closed interval $5 \le x \le 8$. Use the line tangent to the graph of f at $x = 5$ to show that $f(7) \le 4$. Use the secant line for the graph of f on $5 \le x \le 8$ to show that $f(7) \ge \frac{4}{3}$.

3. Consider the differential equation $\frac{dy}{dx} = \frac{(x + 1)(y + 1)}{2}$

(A) On the axes provided below, sketch a slope field for the given differential equation at the 20 points indicated, and sketch the solution curve that passes through the point (0, 1).

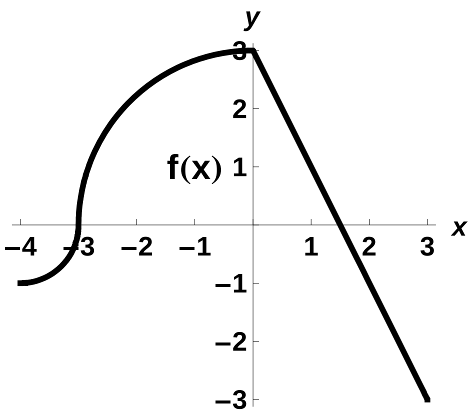


(B) Using the differential equation above, $\frac{dy}{dx} = \frac{(x + 1)(y + 1)}{2}$, and Euler's Method starting at $f(-1) = 1$, and a step size (Δx) of 1, calculate $f(2)$ (take three steps). Plot these points on the graph below, and connect them with a series of line segments



(C) Now, solve the differential equation $\frac{dy}{dx} = \frac{(x + 1)(y + 1)}{2}$ where $y(-1) = 1$

4. The function f is defined on the interval $-4 \le x \le 3$. The graph of f consists of two quarter circles and one line segment, as shown in the figure below. Let $g(x) = 2x + \int_0^x f(t)\,dt$



(A) Find $g(-3)$. Find $g'(x)$ and evaluate $g'(-3)$.

(B) Determine the x – coordinate of the point at which g has an absolute maximum on the interval $-4 \le x \le 3$. Justify your answer.

(C) Find all values of x on the interval $-4 < x < 3$ for which the graph of g has a point of inflection. Give a reason for your answer.