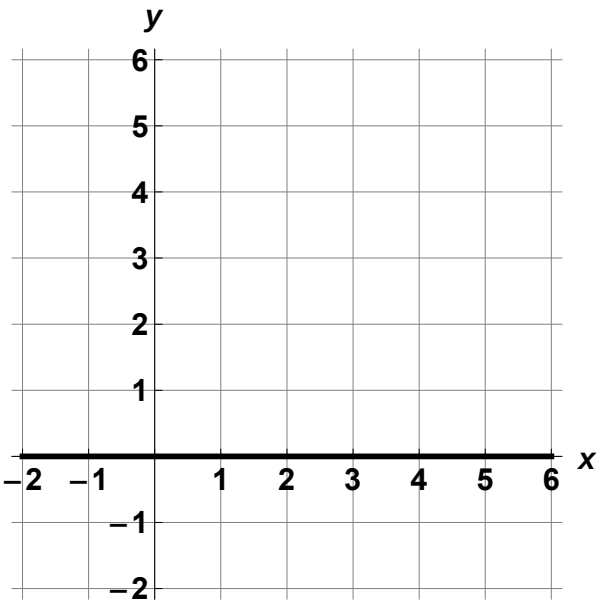
Free Response Final 2014 **Name**

1. Consider $f(x) = \frac{1}{2}x^{\frac{2}{3}}(5 - x)$

(A) Find $f'(x)$, **and use the first derivative test to find all local extrema (x – values only)**

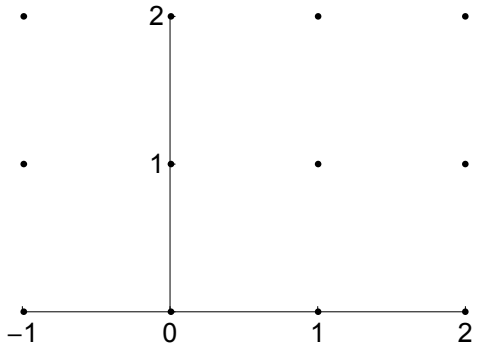
(B) Find $f''(x)$, **and find all points of inflection (x – values only)**

(C) Use the information found in (A) and (B) above to draw a <u>detailed</u> graph of $f(x)$ **below. Be sure to plot roots, extrema, and points of inflection.**



2. Consider the differential equation $\frac{dy}{dx} = 2x - y$

(A) On the axes below, sketch a slope field for the given differential equation at the 12 points indicated, and sketch the solution curve that passes through the point $(0, 1)$.



(B) The solution curve that passes through the point $(0, 1)$ has a local minimum at $x = \ln\left(\frac{3}{2}\right)$. What is the y – coordinate of this local minimum?

(C) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(0) = 1$. Use Euler's Method, starting at $x = 0$ with two steps of equal size, to approximate $f(-0.4)$. Be sure to show your work.

(D) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Determine whether the approximation found in part (C) is less than or greater than $f(-0.4)$. Justify your answer.

3. Let $f(x)$ be the function defined for $x > 0$, with $f(e) = 2$ and $f'(x)$, the first derivative of $f(x)$, given by $f'(x) = x^2 \ln x$

(A) Write an equation for the line tangent to the graph of $f(x)$ at the point $(e, 2)$.

(B) Is the graph of $f(x)$ concave up or concave down on the interval $1 < x < 3$? Justify your answer.

(C) Use antidifferentiation to find $f(x)$.

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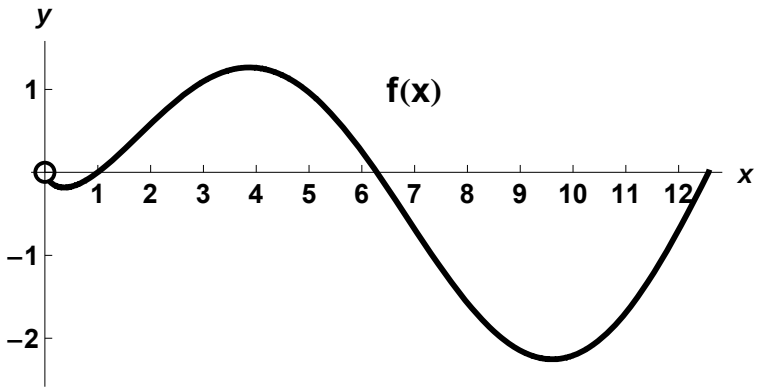

Let $f(x)$ be the function given by $f(x) = (\ln x)\left(\sin\left(\frac{x}{2}\right)\right)$. The figure above shows the graph of $f(x)$ for $0 < x \le 4\pi$. The function $g(x)$ is defined by $g(x) = \int_{2\pi}^{x} f(t)\,dt$ for $0 < x \le 4\pi$

(A) Find $g(2\pi)$ and $g'(2\pi)$

(B) On what intervals, if any, is g decreasing? Justify your answer.

(C) For $0 < x \le 4\pi$, find the value(s) of x at which g has a local extrema. Be sure to indicate min or max, and show your work.

(D) For $0 < x \le 4\pi$, is there a value of x at which the graph of g is tangent to the $x-$axis? Be sure to justify your answer.