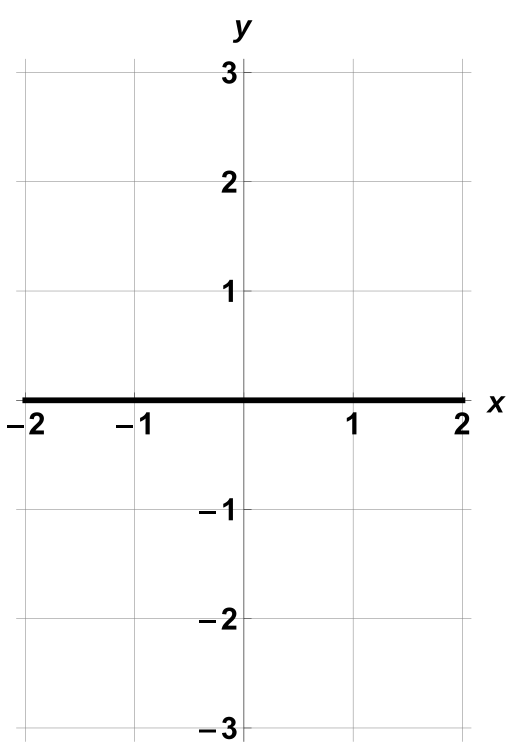
Free Response Final 2015 **Name**

1. Consider $f(x) = x^{\frac{1}{3}}(x^2 - 1)$

(A) Find $f'(x)$, and use the first derivative test to find all local extrema (x – values only)

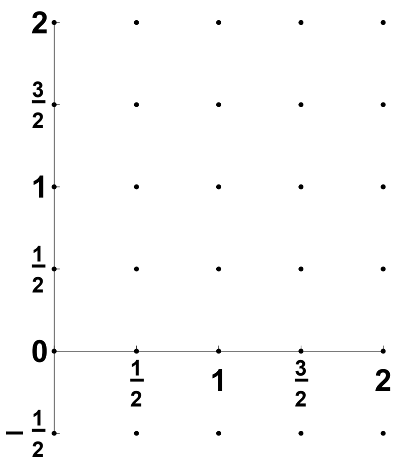
(B) Find $f''(x)$, and find all points of inflection (x – values only)

(C) Use the information found in (A) and (B) above to draw a <u>detailed</u> graph of $f(x)$ below. Be sure to plot roots, extrema, and points of inflection. Also, determine what type of "special" point exists at $x = 0$.



2. Consider the differential equation $\dfrac{dy}{dx} = \dfrac{y}{x} + 1$

(A) On the axes below, sketch a slope field for the given differential equation at the points to the right of the y – axis (no need to plug in $x = 0$, since you would be dividing by 0), and sketch the solution curve that passes through the point $\left(1, \dfrac{1}{2}\right)$.



(B) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(1) = \dfrac{1}{2}$. Use Euler's Method, starting at $x = 1$ with two steps of equal size, to approximate $f(1.4)$.

(C) Verify that the function $y = x \ln x + \dfrac{1}{2} x$ is a solution to the given differential equation. Show all of your work.

(D) Find $\dfrac{d^2 y}{dx^2}$ in terms of x and y. Determine whether the approximation found in part (B) is less than or greater than $f(1.4)$. Justify your answer.

3. Consider the function $y = \sin(\pi x) - x^3 + 4x$

(A) Find the 2 non – negative zeros for the given function. You may want to plot some points and draw a quick sketch (it's okay if you find these zeros by guess and check).

(B) Approximate the area under the curve from the first zero to the second zero, using trapezoids and 4 equal subintervals.

(C) Find the area under the curve from the first zero to the second zero (using calculus, of course).

4. Let $f(x)$ be a differentiable function, defined for all real numbers x, with the following properties

I. $f'(x) = ax^2 + bx$, where a and b are real numbers

II. $f'(1) = 6$ and $f''(1) = 18$

III. $\int_1^2 f(x)\,dx = 18$

(A) Solve for a and b. Show your work.

(B) Find $f(x)$. Show your work.

(C) Using the values you found in part (A) above, evaluate $\int_2^{-1} f'(x)\,dx$