Free Response Final 2016 **Name**

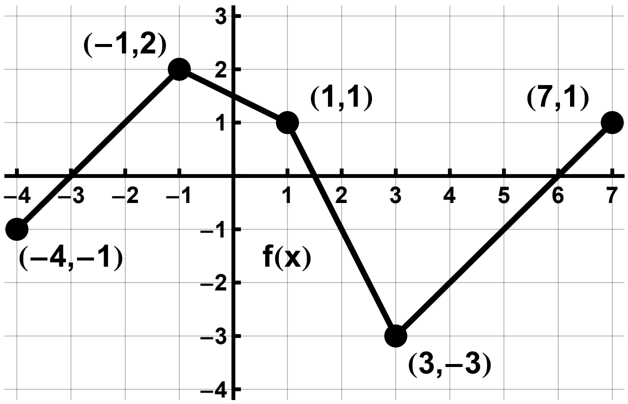
1. Consider $f(x) = (1 - x^2)^{\frac{1}{3}}$

(A) Find $f'(x)$ and use the First Derivative Test to find all local extrema (indicate maxes or mins, and x – values only)

(B) Determine if and where there are any Vertical Tangents and/or Cusps (you can do it!)

(C) Find $f''(x)$ (and simplify). Determine if there are any points of inflection (x – values only)

2. The continuous function f is defined on the interval $-4 \le x \le 7$. The graph of f, which consists of four line segments, is shown in the figure below. Let g be the function given by $g(x) = \frac{1}{2}x + \int_{-1}^{x} f(t)\,dt$



(A) Find $g(-4)$ and $g(6)$

(B) Find $g'(x)$ in terms of $f(x)$. For each of $g''(-1)$ and $g''(5)$, find the value or state that it does not exist.

(C) On what intervals, if any, is the graph of g concave down? Give a reason for your answer.

(D) The function $h(x) = g\left(x^2 + 2\right)$. Find $h'(-1)$, and show the work that leads to your answer.

3. Evelyn is planning on reading for pleasure during the break, starting on Friday. For 16 hours straight, she plans to do nothing but read. Evelyn's reading rate, measured in pages per hour, is represented by the function $r(t)$. Values for $r(t)$ at various times during the day are given in the table below. You may assume that $r(t)$ is differentiable and decreasing.

time (hour)	8 am	10 am	1 pm	3 pm	4 pm	5 pm	8 pm	10 pm	12 am
rate (pages per hour)	32	31	29	28	26	24	22	21	17

(A) Use the data in the table to estimate the instantaneous rate of change of $r(t)$ at 10 pm. Then use this estimation to find the Linearization formula for $r(t)$, $L(t)$, at 10 pm. Assuming that Evelyn decides to read beyond 12 am, use it to approximate her reading rate at 1 am (3 hours after 10 pm).

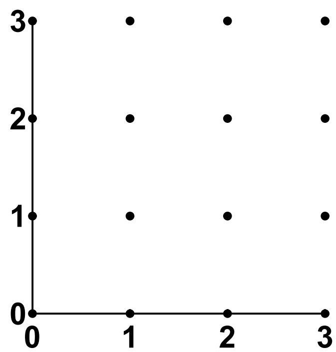
(B) Estimate $\int_0^{16} r(t)\,dt$ (where 0 corresponds to the starting time of 8 am), using a trapezoidal approximation method with 8 subintervals (not the Trapezoidal Rule). Do you think this approximation is an underestimate or overestimate for $\int_0^{16} r(t)\,dt$? Justify your answer.

(C) Find the average rate of change of $r(t)$, indicating the correct units, over the interval from 8 am to 12 am

4. Consider the differential equation $\frac{dy}{dx} = \left(1 - \frac{2}{x^2}\right)(y - 2)$ where $x \neq 0$. Let $y = f(x)$ be the particular solution to the differential equation with initial condition $f(2) = 3$

(A) Find the equation of the normal line to the graph of $f(x)$ at the point $(2, 3)$

(B) On the axes provided, sketch a slope field for the given differential equation for the points indicated (you may skip the points on the y – axis, since the derivative does not exist at $x = 0$).



(C) Find the particular solution $y = f(x)$ to the differential equation $\frac{dy}{dx} = \left(1 - \frac{2}{x^2}\right)(y - 2)$ with initial condition $f(2) = 3$