1. If $f(x) = e^{2x}\tan^{-1}(x)$, then $f'(1) =$

(A) $\frac{e^2}{2}$ (B) $\frac{e^2\pi}{4}$ (C) e^2 (D) $\frac{e^2\pi}{2}$ (E) $\frac{e^2(\pi + 1)}{2}$

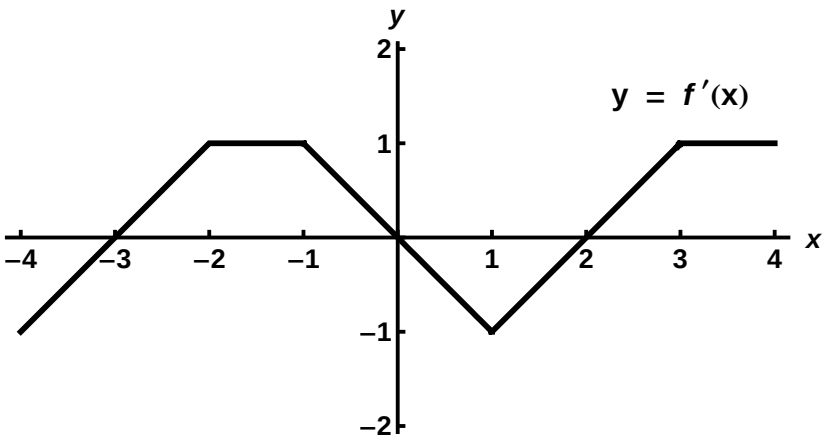
2. The slope of the line tangent to the curve $3x^2 - 2xy + y^2 = 11$ at the point $(1, -2)$ is

(A) $\frac{-1}{6}$ (B) 0 (C) 1 (D) $\frac{5}{3}$ (E) 10

3. If the function $f(x)$ is continuous for all real numbers and $\lim_{x \to a^-} f(x) = B$, then which of the following statements must be true?

I. $\lim_{x \to a^+} f(x) = B$ II. $\lim_{x \to a} f(x) = B$ III. $f(a) = B$

(A) I only (B) II only (C) I and II only (D) III only (E) I, II, and III



For problems 4 – 6, refer to the graph of $y = f'(x)$, the derivative of f, shown above. The graph consists of five line segments, two of which are horizontal.

4. At $x = 1$, f has a

(A) point of discontinuity (B) point of inflection (C) point of nondifferentiabiliy
(D) local maximum (E) local minimum

5. Over the interval $-4 < x < 4$, how many local minima does f have?

(A) One (B) Two (C) Three (D) Four (E) Five

6. If $f(2) = 1$, what is the value of $f(-2)$?

(A) $\frac{-3}{2}$ (B) $\frac{-1}{2}$ (C) $\frac{1}{2}$ (D) $\frac{3}{2}$ (E) $\frac{5}{2}$

7. Provide the most accurate approximation for the following definite integral: $\int_{-4}^{0}\left(4 - \sqrt{16 - x^2}\right) dx$

(A) 3 (B) $\frac{7}{2}$ (C) 4 (D) $\frac{9}{2}$ (E) $\frac{11}{2}$

8. $\lim_{n \to \infty} \sum_{k=1}^{n} \left(1 + \frac{2k}{n}\right)^2 \left(\frac{6}{n}\right) =$

(A) 8 (B) 9 (C) 16 (D) 21 (E) 26

9. The displacement $s(t)$ (in meters) of a particle moving on the x – axis is a function of time t (in seconds). The table below shows this displacement for time values on the interval $[0, 8]$

t (seconds)	0	2	3	5	6	7	8
$s(t)$ (meters)	7	9	12	18	14	12	9

Using the table above, provide the best approximation for the particle's speed at $t = 6$ seconds.

(A) $-4 \dfrac{\text{meters}}{\text{second}}$ (B) $-3 \dfrac{\text{meters}}{\text{second}}$ (C) $-2 \dfrac{\text{meters}}{\text{second}}$ (D) $3 \dfrac{\text{meters}}{\text{second}}$ (E) $4 \dfrac{\text{meters}}{\text{second}}$

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10. Given that $f(-3) = 4$ and $f'(-3) = 2$, which of the following is the tangent line approximation of $f(-3.1)$?

(A) 3.8 (B) 3.9 (C) 4.0 (D) 4.1 (E) 4.2

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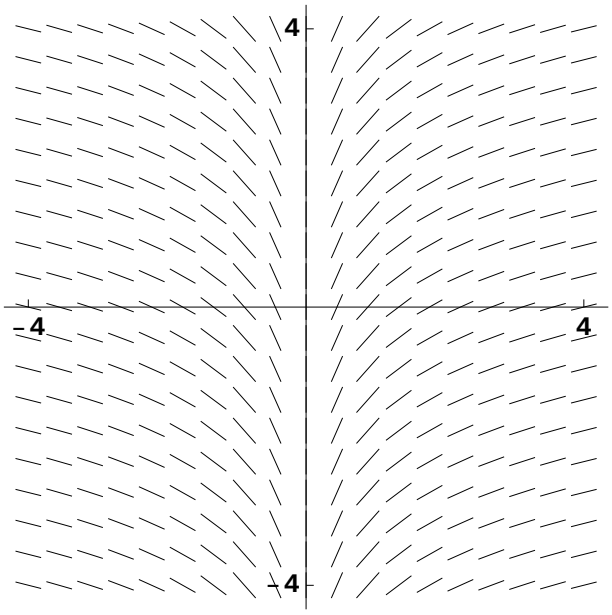
11. The slope field below matches which differential equation?



(A) $\dfrac{dy}{dx} = \dfrac{1}{x}$ (B) $\dfrac{dy}{dx} = \dfrac{1}{x^2}$ (C) $\dfrac{dy}{dx} = \dfrac{y}{x}$ (D) $\dfrac{dy}{dx} = \dfrac{\ln x}{x}$ (E) $\dfrac{dy}{dx} = \dfrac{\sin x}{x}$

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12. $\lim\limits_{x \to 0} x \csc(2x) =$

(A) $-\infty$ (B) $\dfrac{1}{2}$ (C) 1 (D) 2 (E) ∞

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13. If the differential equation $\dfrac{dy}{dx} = y - 2y^2$ has the solution curve $y = f(x)$ containing the point $\left(0, \dfrac{1}{4}\right)$, then $\lim\limits_{x \to \infty} f(x) =$

(A) 0 (B) $\dfrac{1}{4}$ (C) $\dfrac{1}{2}$ (D) 2 (E) The limit does not exist.

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14. If $f(x_1) + f(x_2) = f(x_1 x_2)$, for all positive real numbers x_1 and x_2, which of the following could define $f(x)$?

(A) $2x$ (B) x^2 (C) e^{3x} (D) $\sin(4x)$ (E) $2 \ln x$

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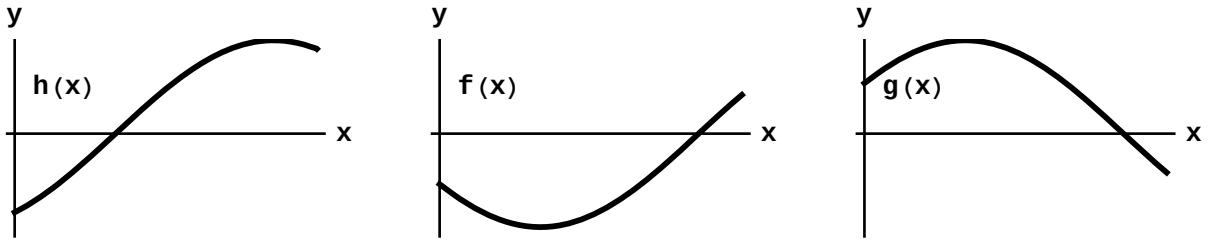
15. The equation of the line tangent to $y = x^3 + 3x^2 + 2$ at its point of inflection is

(A) $y = -6x - 6$ (B) $y = -3x + 1$ (C) $y = 2x + 10$ (D) $y = 3x - 1$ (E) $y = 4x + 1$

16. Given the graphs of $h(x)$, $f(x)$, and $g(x)$ the following could be true



(A) $h'(x) = f(x)$ and $h''(x) = g(x)$ (B) $h'(x) = g(x)$ and $h''(x) = f(x)$
(C) $f'(x) = g(x)$ and $f''(x) = h(x)$ (D) $f'(x) = h(x)$ and $f''(x) = g(x)$
(E) $g'(x) = f(x)$ and $g''(x) = h(x)$

17. The side of a cube is expanding at a constant rate of 2 centimeters per second. What is the instantaneous rate of change of the surface area of the cube, in cm^2 per second, when its volume is 27 cubic centimeters?

(A) 6 (B) 24 (C) 36 (D) 54 (E) 72

18. The efficiency of a motor scooter engine is given by the continuous function $E(c)$, where E is measured in gallons/mile and c is measured in miles. What are the units of $\int_0^{10} E(c)\,dc$?

(A) miles (B) gallons (C) gallon – miles (D) $\frac{\text{gallons}}{\text{mile}}$ (E) $\frac{\text{miles}}{\text{gallon}}$

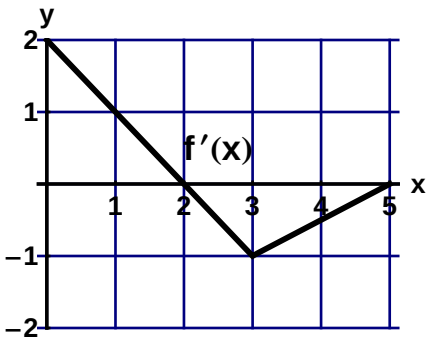
19. If $f(x) = e^x$, which of the following is equal to $f'(e)$?

(A) $\lim_{h\to 0} \frac{e^{x+h}}{h}$ (B) $\lim_{h\to 0} \frac{e^{x+h} - e^e}{h}$ (C) $\lim_{h\to 0} \frac{e^{e+h} - e^e}{h}$ (D) $\lim_{h\to 0} \frac{e^{x+h} - 1}{h}$ (E) $\lim_{h\to 0} \frac{e^{x+h} - e}{h}$

20. Given that $F'(x) = f(x)$, the value of $\int_a^b x f(x^2)\,dx$ is

(A) $\frac{F(b^2) - F(a^2)}{2}$ (B) $bF(b^2) - aF(a^2)$ (C) $2F\left(\sqrt{b}\right) - 2F\left(\sqrt{a}\right)$
(D) $\frac{b^2F(b^2) - a^2F(a^2)}{2}$ (E) $2F(b^2) - 2F(a^2)$

21. Consider the graph of f' below. If $f(5) = -1$, then $f(0) =$



(A) – 1.5 (B) – 1 (C) – 0.5 (D) 0 (E) 0.5

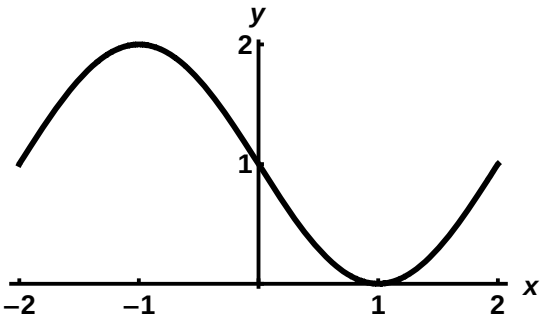
22. Which of the functions given below has an average value of 0 on the interval $[-a, a]$, where $a > 0$?

(A) $\sqrt{x}$ (B) $|x|$ (C) $x^2 + x^4$ (D) $\cos x$ (E) $\sin x$

23. If $\frac{dy}{dx} = 1 - \frac{x}{y}$ and $y(1) = 1$, then when Euler's method with a step size of 0.5 is used to approximate $y(2)$, the approximation is

(A) 0 (B) 0.375 (C) 0.5 (D) 0.75 (E) 1.5

24. Given the graph of $y = f(x)$ shown below, which of the following values is the largest?



(A) $f(0)$ (B) $f'(0)$ (C) $\lim_{h \to 0} \frac{f(h) - 1}{h}$ (D) $\frac{f(1) - f(-1)}{2}$ (E) $\frac{f'(1) - f'(-1)}{2}$

25. Which of the following functions satisfies $0 < f''(x) < f'(x) < f(x)$ for all x?

(A) $f(x) = e^{-x}$ (B) $f(x) = e^{\frac{x}{2}}$ (C) $f(x) = e^{x}$ (D) $f(x) = e^{2x}$ (E) $f(x) = e^{x^2}$

26. Suppose $L(x)$ is the linearization formula for $f(x) = \log_2(x^2)$ at $x = 4$. Which of the following is true?

(A) $L(4.1)$ overestimates $f(4.1)$, and $L(3.9)$ overestimates $f(3.9)$
(B) $L(4.1)$ underestimates $f(4.1)$, and $L(3.9)$ underestimates $f(3.9)$
(C) $L(4.1)$ overestimates $f(4.1)$, and $L(3.9)$ underestimates $f(3.9)$
(D) $L(4.1)$ underestimates $f(4.1)$, and $L(3.9)$ overestimates $f(3.9)$
(E) None of the above are true

27. $\lim_{h \to 0} \left(\frac{1}{h} \int_{1}^{1+h} e^{-t^2} dt \right) =$

(A) $\frac{-1}{2e}$ (B) $\frac{-2}{e}$ (C) 0 (D) $\frac{1}{e}$ (E) The limit does not exist

28. $D_x \left(\int_{-2}^{3} e^{x} \cos(2x)\, dx \right) =$

(A) 0 (B) $e^{x} \cos(2x)$ (C) $e^{3} \cos(6) - e^{-2} \cos(-4)$ (D) $5e^{x} \cos(2x)$ (E) $\frac{e^{x}}{5}(\cos(2x) + 2\sin(2x))$

29. If $g(x) = x|x|$, then $g'(x) =$

(A) $\frac{x^2|x|}{3}$ (B) $2|x|$ (C) x (D) $2x$ (E) $|x|$

30. If $\int_{0}^{3} e^{\sin x} dx = k$, then $\int_{1}^{2} x e^{\sin(4 - x^2)} dx =$

(A) $\frac{-k}{2}$ (B) $\frac{-k}{3}$ (C) $\frac{k}{6}$ (D) $\frac{k}{3}$ (E) $\frac{k}{2}$