1. The function $f(x)$ is twice differentiable, and the graph of $f(x)$ has no points of inflection. If $f(6) = 3$, $f'(6) = \frac{-1}{2}$, and $f''(6) = -2$, which of the following could be the value of $f(7)$?

(A) 2 (B) 2.5 (C) 2.9 (D) 3 (E) 4

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2. Consider a function $f(x)$, where $f(x) > 0$ on $[a, b]$, $f'(x) > 0$ on $[a, b]$, and $f''(x) < 0$ on $[a, b]$ If the interval $[a, b]$ is split into 4 equal subdivisions, and L is the left – hand Riemann sum for the partitioning, R is the right – hand Riemann sum for the partitioning, and T is the trapezoidal approximation for the partitioning, then

(A) $L < T < R$ (B) $T < R < L$ (C) $R < T < L$ (D) $L < R < T$ (E) $R < L < T$

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3. Evaluate $\displaystyle\int_1^2 \frac{x + 1}{x^2 + 2x}\,dx$

(A) $\ln 8 - \ln 3$ (B) $\frac{\ln 8 - \ln 3}{2}$ (C) $\ln 8$ (D) $\frac{3 \ln 2}{2}$ (E) $\frac{3 \ln 2 + 2}{2}$

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4. The base of a solid is the area in the first quadrant bounded by the x – axis, the y – axis, and the circle $x^2 + y^2 = 4$. Every cross section of the solid is an equilateral triangle, perpendicular to the x – axis, with one side in the xy – plane. Find the volume of the solid $\left(\text{area of an equilateral triangle is } \frac{\sqrt{3}}{4} s^2\right)$.

(A) $\frac{16\pi}{3}$ (B) $\frac{4}{3}$ (C) $\frac{16}{3}$ (D) $\frac{4\sqrt{3}}{3}$ (E) $2\sqrt{3}$

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5. If $y = f\left(x^3\right)$ and $f'(x) = \sqrt{3x - 2}$ then $\frac{dy}{dx} =$

(A) $\sqrt{9x^2 - 2}$ (B) $x^3\sqrt{3x^3 - 2}$ (C) $3x^2\sqrt{9x^2 - 2}$

(D) $3x^2\sqrt{3x^3 - 2}$ (E) $18x^3\sqrt{9x^2 - 2}$

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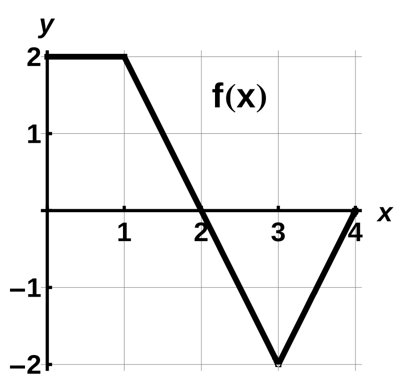
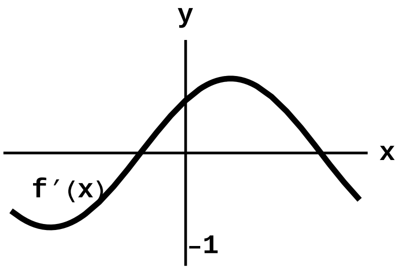
6. Consider the graph below of $f(x)$, defined on the interval $[0, 4]$. Let $G(x) = \displaystyle\int_0^x f(t)\,dt$ and $H(x) = \displaystyle\int_2^x f(t)\,dt$. Which of the statements below is true?



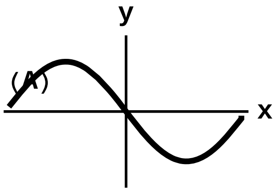
(A) $G(x) = H(x)$ (B) $G'(x) = H'(x + 2)$ (C) $G(x) = H(x) + 3$

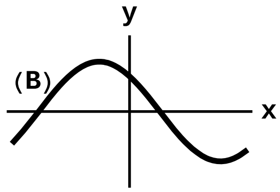
(D) $G(x) = H(x) - 2$ (E) $G(x) = H(x + 2)$

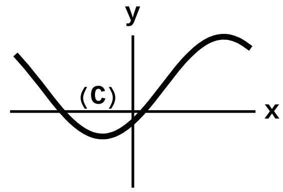
7. Suppose that the <u>derivative</u> of f has the graph as shown below

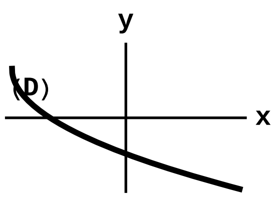


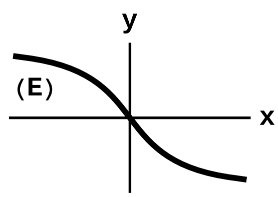
Which of the following could be the graph of f?











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8. If f is a linear function and $0 < a < b$, then $\int_a^b f''(x)\,dx =$

(A) 0 (B) 1 (C) $\frac{ab}{2}$ (D) $b - a$ (E) $\frac{b^2 - a^2}{2}$

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9. Suppose that $f(x)$, $f'(x)$, and $f''(x)$ are continuous for all real numbers x, and that f has the following properties

(i) f is positive on $(-\infty, 4)$ and negative on $(4, \infty)$

(ii) f is increasing on $(-\infty, -1)$ and decreasing on $(-1, \infty)$

(iii) f is concave down on $(-\infty, 2)$ and concave up on $(2, \infty)$

Of the following, which has the greatest numerical value?

(A) $f(5)$ (B) $f''(1)$ (C) $f'(0)$ (D) $f'(-2)$ (E) Not enough information is given

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10. If the base b of a triangle is increasing at a rate of 3 inches per minute while its height h is decreasing at a rate of 3 inches per minute, which of the following must be true about the area A of the triangle?

(A) A is always increasing (B) A is always decreasing (C) A remains constant
(D) A is decreasing only when $b > h$ (E) A is decreasing only when $b < h$

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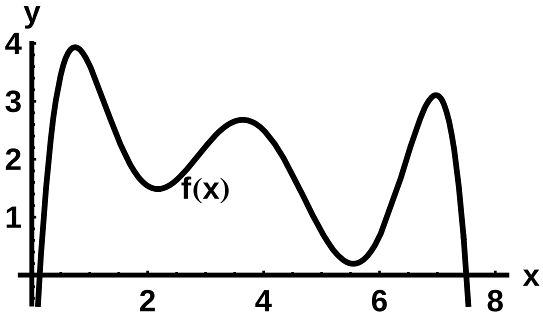
11. On the interval $\left(0, \frac{\pi}{2}\right)$, if $y = (\sin x)^x$, then $\frac{dy}{dx} =$

(A) $x \ln(\sin x)$ (B) $(\sin x)^x \cot x$ (C) $x(\sin x)^{x-1}(\cos x)$
(D) $(\sin x)^x(x\cos x + \sin x)$ (E) $(\sin x)^x(x\cot x + \ln(\sin x))$

12. If the curve below represents the complete polynomial graph of $f(x)$, then the graph of $f''(x)$ will cross the x – axis in how many points?



(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

13. Evaluate $\int \sin^{-1} x \, dx$

(A) $x \sin^{-1} x + \sqrt{1 - x^2} + C$ (B) $x \sin^{-1} x - \sqrt{1 - x^2} + C$ (C) $x \sin^{-1} x + \frac{1}{2}\sqrt{1 - x^2} + C$

(D) $x \sin^{-1} x - 2\sqrt{1 - x^2} + C$ (E) $x \sin^{-1} x - \frac{1}{2}\sqrt{1 - x^2} + C$

14. Which of the following is a critical value for the function $f(x) = (2x + 3)^3 (4x + 3)^3$? This would be a root, or zero, from the first derivative.

(A) $x = \frac{-17}{12}$ (B) $x = \frac{-9}{8}$ (C) $x = -1$ (D) $x = \frac{-9}{10}$ (E) $x = \frac{17}{12}$

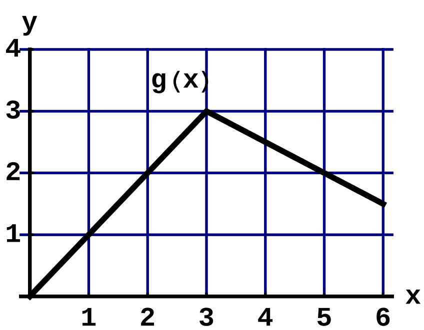
15. If $f'(x) = 3x^2$ and $f(-2) = 1$, then find $\int_0^2 f(x)\, dx$

(A) 28 (B) 22 (C) 16 (D) 9 (E) 4

16. An equation of the normal line to the graph of $f(x) = x^3 + 3x^2 + 7x - 1$ at the point where $x = -1$ is

(A) $x + 4y = -25$ (B) $x + 4y = 25$ (C) $4x - y = 2$
(D) $x - 4y = 23$ (E) $4x + y = -10$

17. Find $D_x\left(\frac{1}{g(x)}\right)$ at $x = 5$ if $g(x)$ is defined by the following graph

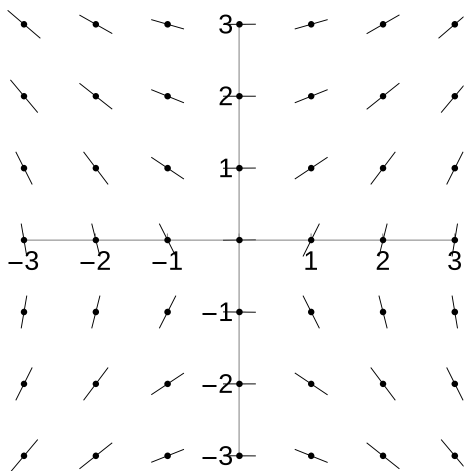


(A) $\frac{-1}{2}$ (B) $\frac{-1}{8}$ (C) $\frac{1}{8}$ (D) $\frac{1}{4}$ (E) $\frac{1}{2}$

18. If $a = be^{2x}$, and $\ln a = 7$ and $\ln b = 3$, then find x

(A) $\ln 7$ (B) 2 (C) $\frac{7}{3}$ (D) 4 (E) e^3

19. The slope field below matches which differential equation?



(A) $\frac{dy}{dx} = \frac{x}{2y}$ (B) $\frac{dy}{dx} = \frac{2y}{x}$ (C) $\frac{dy}{dx} = \frac{1}{3}x^2 + 1$

(D) $\frac{dy}{dx} = \frac{x}{y + \frac{1}{2}}$ (E) $\frac{dy}{dx} = \frac{1}{xy}$

20. Suppose that $f''(x) > 0$ throughout the domain of a given function. If we are given $\frac{dy}{dx}$, an initial value, and asked to use Euler's Method to approximate a solution, then

(A) Euler's Method will underapproximate the solution
(B) Euler's Method will overapproximate the solution
(C) Euler's Method will underapproximate the solution for all $x > 0$, overapproximate for all $x \le 0$
(D) Euler's Method will overapproximate the solution for all $x > 0$, underapproximate for all $x \le 0$
(E) Not enough information is given to make this determination

21. Let f be the function defined by $f(x) = x^3 + x$. If $g(x) = f^{-1}(x)$ and $g(2) = 1$, then find the value of $g'(2)$.

(A) $\frac{1}{13}$ (B) $\frac{1}{4}$ (C) $\frac{7}{4}$ (D) 4 (E) 13

22. Find the slope of the line tangent to the curve $3y^2 - 2x^2 = 6 - 2xy$ at the point $(3, 2)$.

(A) 0 (B) $\frac{4}{9}$ (C) $\frac{7}{9}$ (D) $\frac{6}{7}$ (E) $\frac{5}{3}$

23. Suppose
$$g(x) = \begin{cases} \dfrac{x^2 + 2x + 1}{x + 1} & x < 1 \\ 2x & x \ge 1 \end{cases}$$
The best description concerning the continuity of $g(x)$ is that the function

(A) is continuous (B) has a jump discontinuity (C) has an infinite discontinuity
(D) has a removable discontinuity (E) has both a removable and infinite discontinuity

24. At each point (x, y) on a certain curve, the slope of the curve is $3x^2y$. If the curve contains the point $(0, 8)$, then the equation of the curve is

(A) $y = 8e^{x^3}$ (B) $y = x^3 + 8$ (C) $y = e^{x^3} + 7$
(D) $y = \ln(x + 1) + 8$ (E) $y^2 = x^3 + 8$

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25. The equation of the line tangent to the curve $y = \dfrac{kx + 8}{k + x}$ at $x = -2$ is $y = x + 4$. The value of k is

(A) -3 (B) -1 (C) 1 (D) 3 (E) 4

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26. For values of h very close to 0, which of the following functions best approximates
$f(x) = \dfrac{\csc(x + h) - \csc x}{h}$?

(A) $\csc x$ (B) $\dfrac{\csc x}{h}$ (C) $-\csc x$ (D) $\dfrac{-\csc x}{h}$ (E) $-\csc x \cot x$

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27. $\lim\limits_{x \to 3} \dfrac{x^2 - 9}{|x - 3|} =$

(A) -6 (B) -1 (C) 1 (D) 6 (E) Does Not Exist

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28. Find $D_x\left(\ln\left(h\left(x^2\right)\right)\right)$ at $x = 3$ if $h(3) = 2$, $h(9) = 4$, $h'(3) = 1$, and $h'(9) = 2$

(A) $\dfrac{1}{4}$ (B) $\dfrac{1}{2}$ (C) $\dfrac{3}{2}$ (D) 2 (E) 3

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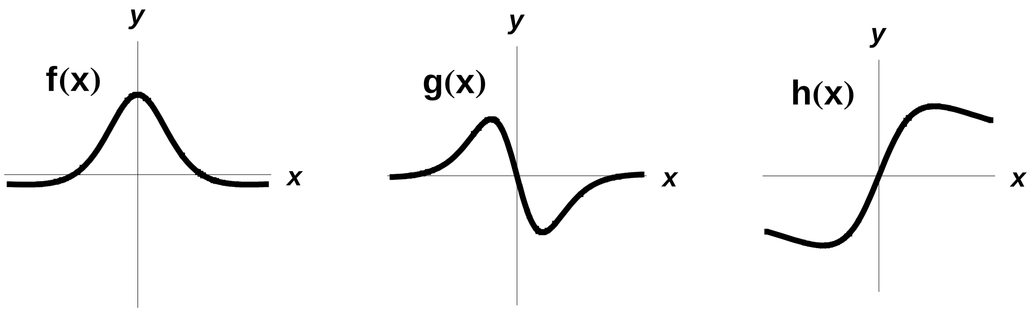
29. Given the graphs of $f(x)$, $g(x)$, and $h(x)$, which could be true?



(A) $f'(x) = g(x)$ and $f''(x) = h(x)$ (B) $f'(x) = h(x)$ and $f''(x) = g(x)$
(C) $h'(x) = g(x)$ and $h''(x) = f(x)$ (D) $h'(x) = f(x)$ and $h''(x) = g(x)$
(E) $g'(x) = h(x)$ and $g''(x) = f(x)$

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30. If $f(x) = \cos x - \sin x$, then which of the following is true on the interval $[0, 2\pi]$?

(A) $f(x)$ has a local maximum at $x = \dfrac{3\pi}{4}$ (B) $f(x)$ has a local minimum at $x = \dfrac{\pi}{4}$

(C) $f(x)$ is increasing on $\left[\dfrac{3\pi}{4}, \dfrac{7\pi}{4}\right]$ only (D) $f(x)$ is decreasing on $\left[\dfrac{3\pi}{4}, \dfrac{5\pi}{4}\right]$ only

(E) $f(x)$ has a local minimum at $x = \dfrac{5\pi}{4}$