1. Which of the following functions satisfies $f''(x) < f'(x) < f(x)$ for all x?

(A) $f(x) = 2^x$ (B) $f(x) = e^x$ (C) $f(x) = 3^x$ (D) $f(x) = 4^x$ (E) $f(x) = e^{x^2}$

2. Evaluate $\lim_{x \to 0} \dfrac{\int_{\sqrt{\pi}+x}^{\sqrt{\pi}} \cos(t^2)\,dt}{x}$

(A) -1 (B) $\dfrac{-\sqrt{3}}{2}$ (C) 0 (D) 1 (E) Does Not Exist

3. If $(f(x_1))(f(x_2)) = f(x_1 + x_2)$ for all positive real numbers x_1 and x_2, which of the following could define $f(x)$?

(A) $2x + 3$ (B) $x^2 + 1$ (C) $\sin(2x)$ (D) 3^x (E) $\ln x$

4. If the line $y = \frac{1}{3}x - \frac{2}{3}$ is tangent to the curve $y = x^{\frac{2}{3}} + C$ then $C =$

(A) -3 (B) -2 (C) -1 (D) 0 (E) 1

5. Evaluate $\int_{-2}^{2} e^{x^2} \sin(2x)\,dx$

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

6. Evaluate $\int_{\pi}^{0} x^2 \cos x\,dx$

(A) -2π (B) $-\pi$ (C) 0 (D) π (E) 2π

7. If $\dfrac{dy}{dx} = 2x^2 y$ then $\dfrac{d^2 y}{dx^2} =$

(A) $4xy$ (B) $4xy + 2x^2$ (C) $4xy + 4x^4 y$ (D) $4xy + 4x^2 y$ (E) $8x^5 y^2$

8. Suppose $g(x) = 4 - x^2$ with domain $(-\infty, 0]$. For the inverse of this function, $g^{-1}(x)$, find $D_x\left(g^{-1}(x)\right)$ at $x = 3$

(A) -1 (B) $\dfrac{-1}{2}$ (C) $\dfrac{-1}{4}$ (D) $\dfrac{1}{2}$ (E) 1

9. Evaluate $\int_{-1}^{3} \sqrt{x^2 - 2x + 1}\,dx$

(A) 0 (B) 2 (C) 4 (D) 5 (E) 6

10. Consider a normal line to a function $f(x)$ at the point $(-2, 3)$. If this normal line also goes through the point $(-5, 4)$, then $f'(-2) =$

(A) -3 (B) -1 (C) $\dfrac{-1}{3}$ (D) $\dfrac{1}{3}$ (E) 3

11. $4\sin\left(x + \frac{2\pi}{3}\right) =$

(A) $2\sqrt{3}\sin x - 2\cos x$ (B) $-2\sin x + 2\sqrt{3}\cos x$ (C) $-2\sqrt{3}\sin x + 2\cos x$

(D) $2\sin x - 2\sqrt{3}\cos x$ (E) $\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x$

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12. Assume that the function $g(x)$ is the inverse of the function $f(x)$. Given the table below, find $g'(2)$

x	-3	-1	2
$f(x)$	-1	2	4
$f'(x)$	3	2	1

(A) -2 (B) $\frac{-1}{2}$ (C) $\frac{1}{2}$ (D) 1 (E) 2

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13. If $\lim_{x \to a} f(x) = L$, then which of the following statements must be true?

I. $f(a) = L$ II. $f(x)$ is continuous at $x = a$

III. $f(x)$ is differentiable at $x = a$

(A) I only (B) I and II only (C) I and III only (D) I, II, and III (E) None of these

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14. Evaluate $\lim_{x \to \frac{-\pi}{6}} \frac{\sec^2(2x) - 4}{x + \frac{\pi}{6}}$

(A) $-16\sqrt{3}$ (B) $-8\sqrt{3}$ (C) $-4\sqrt{3}$ (D) $-2\sqrt{3}$ (E) $-\sqrt{3}$

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15. The function $f(x) = \begin{cases} 4 - x^2, & x \le 1 \\ ax + b, & x > 1 \end{cases}$ is continuous and differentiable for all real numbers

What must be the values of a and b?

(A) $a = 2, b = 1$ (B) $a = -2, b = 5$ (C) $a = 2, b = 5$

(D) $a = -2, b = 1$ (E) No values for a and b will make this a continuous and differentiable function

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16. Evaluate $\int_0^5 \frac{1}{\sqrt{3x + 1}}\,dx$

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) 1 (D) 2 (E) 6

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17. Find the slope of the normal line to the curve $\sqrt{x^2 y} - xy^2 = 4$ at the point $(-2, 1)$

(A) $\frac{-5}{2}$ (B) $\frac{-3}{2}$ (C) $\frac{-1}{2}$ (D) $\frac{2}{3}$ (E) $\frac{2}{5}$

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18. The rate at which a certain calculus student can do multiple choice problems is $P(t)$, where $P(t)$ is measured in problems per minute and t is measured in minutes. What are the units of $\int_0^{60} P(t)\,dt$?

(A) $\frac{\text{problems}}{\text{minute}}$ (B) minutes (C) $\frac{\text{minutes}}{\text{problem}}$ (D) problems (E) headaches

19. If $f(x) = \dfrac{x+1}{x}$ on the domain $\left[\dfrac{1}{2}, 2\right]$ then find the number c on the interval $\left(\dfrac{1}{2}, 2\right)$ that satisfies the Mean Value Theorem

(A) 1 (B) $\dfrac{6}{5}$ (C) $\dfrac{5}{4}$ (D) $\dfrac{4}{3}$ (E) $\dfrac{3}{2}$

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20. If $f(x) = x^3 |x|$ then $f'(x) =$

(A) $-4x^3$ (B) $4|x^3|$ (C) $4x^3$ (D) $3x^2|x|$ (E) $-3x^2|x|$

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21. Let $y = f(x)$ be the solution to the differential equation $\dfrac{dy}{dx} = x^2 y + 1$ with the initial condition $f(1) = -2$. What is the approximation for $f(3)$ if Euler's Method is used, starting at $x = 1$ with 2 steps of equal size?

(A) -20 (B) -16 (C) -14 (D) -13 (E) -12

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22. Find the average value of $f(x) = |\cos x|$ on the interval $[0, 2\pi]$

(A) $\dfrac{1}{\pi}$ (B) $\dfrac{2}{\pi}$ (C) $\dfrac{9}{4\pi}$ (D) $\dfrac{5}{2\pi}$ (E) 1

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23. Evaluate $\lim\limits_{x \to 0}\left(\dfrac{4}{x^2} - \dfrac{3}{x^3} + \dfrac{2}{x^4}\right)$

(A) $-\infty$ (B) 0 (C) 1 (D) ∞ (E) Does Not Exist

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24. If the position of a particle on a number line is given by the function $s(t) = 2t^{\frac{3}{2}} - 3t^2 + 4$ on the interval $[1, 9]$, then find the average acceleration of the particle on the given interval

(A) $\dfrac{-21}{4}$ (B) $\dfrac{-17}{4}$ (C) $\dfrac{-9}{4}$ (D) $\dfrac{-5}{4}$ (E) $\dfrac{9}{4}$

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25. For what value(s) of x will the graphs of the tangent lines of $f(x) = 2\ln|3x|$ and $g(x) = \dfrac{3}{2}x^2 - x$ be parallel?

(A) $\dfrac{2}{3}$ and 1 (B) -1 and $\dfrac{2}{3}$ (C) $\dfrac{2}{3}$ (D) 2 (E) $\dfrac{-2}{3}$ and 1

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26. Consider the functions $f(x) = e^{-x}$ and $g(x) = 4e^x$. The y-value for the intersection point of the graphs of these two functions is $y =$

(A) 2 (B) $2\ln 3$ (C) $3\ln 2$ (D) $4\ln 2$ (E) $5\ln 2$

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27. If $f(x) = \dfrac{1}{2}\sin 2x - \sin x$, then which of the following is true on the interval $[0, 2\pi]$?

(A) $f(x)$ has a local minimum at $x = \dfrac{\pi}{3}$

(B) $f(x)$ has a local minimum at $x = \dfrac{5\pi}{6}$

(C) $f(x)$ has a local minimum at $x = \pi$

(D) $f(x)$ is decreasing on $\left[0, \dfrac{2\pi}{3}\right]$ only

(E) $f(x)$ is increasing on $\left[\dfrac{2\pi}{3}, \dfrac{4\pi}{3}\right]$ only

28. Considering the table of values below, find $\dfrac{d}{dx}\left(\dfrac{3}{h(x)}\right)$ at $x = 2$

x	0	2	3
h(x)	– 1	– 3	– 4
h′(x)	– 2	– 1	0

(A) $\dfrac{-2}{3}$ (B) $\dfrac{-1}{3}$ (C) $\dfrac{-1}{9}$ (D) $\dfrac{1}{3}$ (E) $\dfrac{2}{3}$

29. Which of the following is a critical value for the function $f(x) = (1 - 3x)^2 (1 - 2x)^2$

(A) $\dfrac{-1}{3}$ (B) $\dfrac{1}{12}$ (C) $\dfrac{5}{12}$ (D) $\dfrac{11}{12}$ (E) $\dfrac{4}{3}$

30. Evaluate $D_x\left(\displaystyle\int_{3x}^{1}\sqrt{1 + (2t)^4}\,dt\right)$

(A) $-3\sqrt{1 + (6x)^4}$ (B) $-\sqrt{1 + (6x)^4}$ (C) $\sqrt{1 + (6x)^4}$

(D) $3\sqrt{1 + (6x)^4}$ (E) $18\sqrt{1 + (6x)^4}$