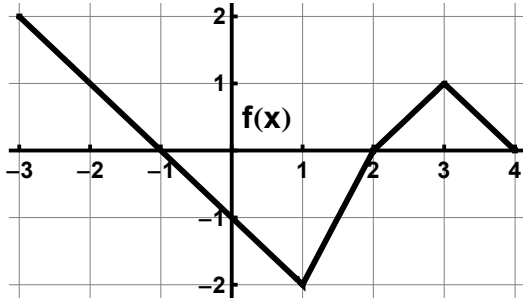


1. If  $y = g(x^5)$  and  $g'(x) = \log_5 x$  then  $\frac{dy}{dx} =$

- (A)  $25x^4 \log_5 x$  (B)  $5x^5 \log_5 x$  (C)  $x^4 \log_5 x^5$  (D)  $25x^4 \log_5 x^5$  (E)  $x \log_5 x^5$

2. The graph of the function  $f(x)$ , shown below, consists of four line segments. If the function  $g(x)$  is an antiderivative of  $f$  such that  $g(-1) = 1$ , for how many values of  $c$ , where  $-3 \leq c \leq 4$ , does  $g(c) = -1$ ?



- (A) Zero (B) One (C) Two (D) Three (E) Four

3. Find the average value of the function  $f(x) = 1 - \sqrt{4 - x^2}$  on the interval  $[-2, 0]$

- (A)  $\frac{1}{2} - \frac{\pi}{2}$  (B)  $\frac{1}{2} - \frac{\pi}{4}$  (C)  $1 - \frac{\pi}{4}$  (D)  $1 - \frac{\pi}{2}$  (E)  $2 - \frac{\pi}{4}$

4. Let  $y = f(x)$  be a twice-differentiable function such that  $f(-2) = 3$  and  $\frac{dy}{dx} = y^2 + y$ . What is

the value of  $\frac{d^2y}{dx^2}$  at  $x = -2$ ?

- (A) 12 (B) 24 (C) 48 (D) 72 (E) 84

5.  $\lim_{h \rightarrow 0} \left( \frac{1}{h} \int_{\sqrt{\pi}}^{\sqrt{\pi}+h} \cos(x^2) dx \right)$

- (A) -1 (B) 0 (C) 1 (D) 2 (E) The limit does not exist

6. The table below gives selected values for a differentiable and decreasing function  $f$  and its derivative. If  $f^{-1}$  is the inverse function of  $f$ , what is the value of  $(f^{-1})'(3)$ ?

$x$	$f(x)$	$f'(x)$
1	37	0
2	3	-7
3	-4	-62

- (A) -62 (B)  $-\frac{1}{7}$  (C)  $-\frac{1}{62}$  (D)  $\frac{1}{62}$  (E)  $\frac{1}{7}$

7. The function  $f$  is continuous on the closed interval  $[-3, 5]$ . If  $\int_{-3}^5 f(x) dx = 18$  and  $\int_5^1 f(x) dx = -14$ ,

then  $\int_1^{-3} 2f(x) dx =$

- (A) -12 (B) -8 (C) -4 (D) 4 (E) 8

8. The function  $y = h(x)$  is differentiable and increasing for all real numbers. On what intervals is the function  $y = h(x^3 + 6x^2)$  increasing?

- (A)  $(-\infty, -4]$  and  $[0, \infty)$       (B)  $[-4, 0]$       (C)  $[-6, \infty)$       (D)  $[-4, \infty)$       (E)  $(-\infty, \infty)$

9. Suppose  $L(x)$  is the linearization formula for  $f(x) = x^3 - 6x^2 + 6x + 1$  at  $x = 2$ . Which of the following is true?

- (A)  $L(1.9)$  underestimates  $f(1.9)$ , and  $L(2.1)$  underestimates  $f(2.1)$   
 (B)  $L(1.9)$  underestimates  $f(1.9)$ , and  $L(2.1)$  overestimates  $f(2.1)$   
 (C)  $L(1.9)$  overestimates  $f(1.9)$ , and  $L(2.1)$  underestimates  $f(2.1)$   
 (D)  $L(1.9)$  overestimates  $f(1.9)$ , and  $L(2.1)$  overestimates  $f(2.1)$   
 (E) None of the above are true

10. The table below gives values of  $f$ ,  $f'$ ,  $g$ , and  $g'$  for selected values of  $x$ . If  $h(x) = f(g(2x))$ , what is the value of  $h'(2)$ ?

$x$	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
-1	-6	3	8	9
2	6	-4	-2	7
4	5	-2	-1	-3

- (A) -24      (B) -18      (C) -9      (D) 9      (E) 18

11. Given that  $F'(x) = f(x)$ , the value of  $\int_a^b \frac{f(\sqrt{x})}{\sqrt{x}} dx$  is

- (A)  $\frac{1}{2}(F(\sqrt{b}) - F(\sqrt{a}))$       (B)  $(F(b) - F(a))$       (C)  $(F(\sqrt{b}) - F(\sqrt{a}))$   
 (D)  $2(F(\sqrt{b}) - F(\sqrt{a}))$       (E)  $4(F(\sqrt{b}) - F(\sqrt{a}))$

12. Find the solution to the differential equation  $\frac{dy}{dx} = 3y^2$  with the initial condition  $y(0) = 5$

- (A)  $y = \frac{5}{1 - 15x}$       (B)  $y = \frac{5}{1 + 15x}$       (C)  $y = \sqrt{e^{3x} + 25}$       (D)  $y = \sqrt{\frac{e^{3x}}{25}}$       (E)  $y = \sqrt{25e^{3x}}$

13. Evaluate  $\lim_{x \rightarrow \frac{\pi}{12}} \frac{\csc(2x) + 2}{x + \frac{\pi}{12}}$

- (A)  $-4\sqrt{3}$       (B)  $-2\sqrt{3}$       (C)  $-\sqrt{3}$       (D)  $\sqrt{3}$       (E)  $4\sqrt{3}$

14. Evaluate  $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left( 3 \left( -1 + \frac{3k}{n} \right)^2 - 2 \left( -1 + \frac{3k}{n} \right) \right) \left( \frac{6}{n} \right)$

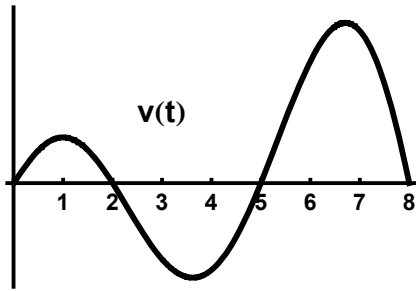
- (A) 2      (B) 3      (C) 6      (D) 12      (E) 24

15. If  $G(x) = \int_1^{\frac{x}{2}} \cot^{-1} t dt$  then  $G'(-2\sqrt{3}) =$

- (A)  $\frac{-\pi}{3}$       (B)  $\frac{-\pi}{6}$       (C)  $\frac{\pi}{12}$       (D)  $\frac{\pi}{6}$       (E)  $\frac{5\pi}{12}$

16. Over the time interval  $0 \leq t \leq 8$ , a particle moves along the  $x$ -axis. The graph of the particle's velocity,  $v$ , is shown below. Over the time interval  $0 \leq t \leq 8$ , the particle's displacement is 4 and the

particle travels a total distance of 11. What is the value of  $\int_2^5 v(t) dt$  ?



- (A) -4 (B) -3.5 (C) -3 (D) -2.5 (E) -2

17. Find the Linearization formula,  $L(x)$ , for the function  $f(x) = 5 + \int_x^{-1} e^{-t^3} dt$  at the point where  $t = -1$

- (A)  $L(x) = 5 + \frac{1}{e}(x - 1)$  (B)  $L(x) = 5 + \frac{1}{e}(x + 1)$  (C)  $L(x) = 5 - \frac{1}{e}(x + 1)$   
 (D)  $L(x) = 5 + e(x + 1)$  (E)  $L(x) = 5 - e(x + 1)$

18. If  $f(x) = \ln x$ , which of the following is equal to  $f'(e)$  ?

- (A)  $\lim_{a \rightarrow 0} \frac{\ln(x+a)}{a}$  (B)  $\lim_{a \rightarrow 0} \frac{\ln(x+a) - \ln x}{a}$  (C)  $\lim_{a \rightarrow 0} \frac{\ln(e+a) - e}{a}$  (D)  $\lim_{a \rightarrow 0} \frac{\ln(e+a) - 1}{a}$  (E)  $\lim_{a \rightarrow 0} \frac{\ln(e+h) - 1}{a}$

19. Evaluate  $\int_0^{\frac{1}{2}} \cos^{-1} x dx$

- (A)  $\frac{\pi}{6}$  (B)  $\frac{\pi}{3}$  (C)  $\frac{\pi}{6} - \frac{\sqrt{3}}{2}$  (D)  $\frac{\pi}{3} - \frac{\sqrt{3}}{2}$  (E)  $\frac{\pi}{6} - \frac{\sqrt{3}}{2} + 1$

20. Let  $f$  be the function given by  $f(x) = \frac{cx}{x^2 + 4}$ , where  $c$  is a constant. For what values of  $c$ , if any, is  $f$  strictly increasing for all  $x$  values on the interval  $(-2, 2)$  ?

- (A)  $c < 0$  (B)  $c = 0$  (C)  $c > 0$  (D)  $c > 1$  only (E) There are no such values of  $c$

21. If the differential equation  $\frac{dy}{dx} = \frac{1}{5}y - \frac{1}{300}y^2$  has the solution curve  $y = f(x)$  containing the point  $(0, 20)$ , then find  $\lim_{x \rightarrow \infty} f(x)$

- (A) 60 (B) 80 (C) 120 (D) 180 (E) The limit does not exist

22. Approximate  $\int_0^3 (9x^2 - 24x + 18) dx$  using three inscribed rectangles of equal width.

- (A) 10 (B) 11 (C)  $\frac{23}{2}$  (D) 12 (E)  $\frac{25}{2}$

23. Evaluate  $\lim_{x \rightarrow 0} \left( \frac{5}{x} + \frac{3}{x^2} - \frac{1}{x^3} \right)$

- (A)  $-\infty$  (B)  $-1$  (C)  $0$  (D)  $\infty$  (E) Does Not Exist

24. Let  $f$  be the function defined below. Which of the following statements about  $f$  are true?

$$f(x) = \begin{cases} 2x - 7 & \text{for } x < 4 \\ \ln(2x - 7) & \text{for } x \geq 4 \end{cases}$$

- I.  $\lim_{x \rightarrow 4^-} f(x) = \lim_{x \rightarrow 4^+} f(x)$       II.  $\lim_{x \rightarrow 4^-} f'(x) = \lim_{x \rightarrow 4^+} f'(x)$   
 III.  $f$  is differentiable at  $x = 4$

- (A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, and III

25. Which of the following functions satisfies  $f'(x) < f''(x) < f(x)$  for all  $x$ ?

- (A)  $2^x$  (B)  $5^x$  (C)  $e^x$  (D)  $2^{-x}$  (E)  $5^{-x}$

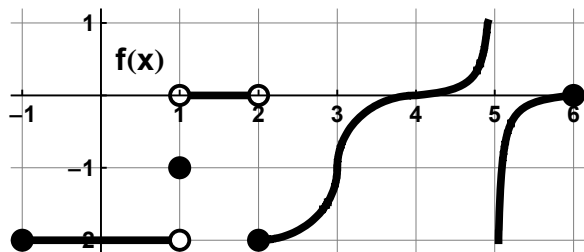
26. If  $f''(x) = (x - 2)(x + 3)^2$  then the graph of  $f$  is concave up for

- (A)  $x < -3$  (B)  $-3 < x < 2$  (C)  $x > 2$  (D)  $x < -3$  and  $x > 2$  (E)  $x < 3$

27. Which of the following is a critical value for the function  $f(x) = (2 + 3x)^2(3 - 2x)^3$ ?

- (A)  $-\frac{2}{7}$  (B)  $\frac{1}{12}$  (C)  $\frac{1}{5}$  (D)  $\frac{3}{7}$  (E)  $8$

28. The graph of the function  $f$  is shown below. For what values of  $x$  does  $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \infty$ ?



- (A) 3 only (B) 1 and 3 (C) 1 and 5 (D) 3 and 5 (E) 1, 3, and 5

29. Let  $f$  be the function defined below, where  $a$  and  $b$  are constants. If  $f$  is differentiable at  $x = \frac{-\pi}{6}$ , then the product  $ab =$

$$f(x) = \begin{cases} 2a \sin x + b & \text{for } x \leq \frac{-\pi}{6} \\ \cos 2x & \text{for } x > \frac{-\pi}{6} \end{cases}$$

- (A)  $-\frac{5}{2}$  (B)  $-\frac{3}{2}$  (C)  $-\frac{1}{2}$  (D)  $\frac{1}{2}$  (E)  $\frac{3}{2}$

30. The slope of the line tangent to the curve  $3y - 2\sqrt{x^2y} - x^2 = 7$  at the point  $(-1, 4)$  is

- (A)  $-\frac{12}{5}$  (B)  $-2$  (C)  $-\frac{4}{5}$  (D)  $\frac{6}{5}$  (E)  $\frac{8}{5}$