1.2 and 1.3 Functions, Graphs, and Exponential Functions

Compositions

$f \circ g = f(g(x))$ → The domain is the intersection of the domain of $g(x)$ and the domain of simplified $f(g(x))$

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Rules for Exponents

$a^x a^y = a^{x+y}$ $\quad \frac{a^x}{a^y} = a^{x-y}$ $\quad (a^x)^y = a^{xy}$ $\quad a^x b^x = (ab)^x$ $\quad \left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$

Growth and Decay Equation

$y = k a^x$

Half – Life

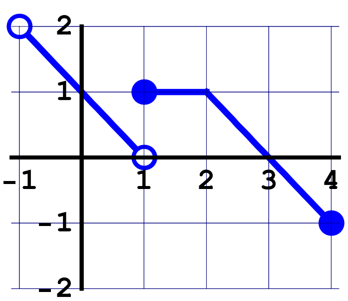
$$y = k\left(\left(\frac{1}{2}\right)^{\frac{t}{\text{life}}}\right) = k\left((2)^{\frac{-t}{\text{life}}}\right)$$ where k is the initial value

Compound Interest

Compounded n times per year → $y = A\left(1 + \frac{k}{n}\right)^{tn}$ A is initial value, k is the rate

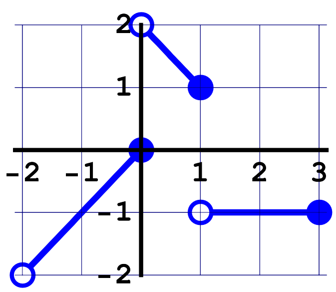
Compounded continuously → $y = A e^{kt}$ A is initial value, k is the rate

1. Write a piecewise formula for the following function.



$$f(x) = \begin{cases} -x + 1 & (-1, 1) \\ 1 & [1, 2] \\ -x + 3 & (2, 4] \end{cases}$$

2. Write a piecewise formula for the following function.



$$f(x) = \begin{cases} x & (-2, 0] \\ -x + 2 & (0, 1] \\ -1 & (1, 3] \end{cases}$$

3. Find $f(g(x))$ and $g(f(x))$ and their domain and range if $f(x) = \sqrt{x + 3}$ and $g(x) = x^2 - 5$

(a) $f(g(x)) = \sqrt{x^2 - 5 + 3} = \boxed{\sqrt{x^2 - 2}}$ $\quad$ D: $\left(-\infty, -\sqrt{2}\right] \cup \left[\sqrt{2}, \infty\right)$ $\quad$ R: $[0, \infty)$

(b) $g(f(x)) = \left(\sqrt{x + 3}\right)^2 - 5 = \boxed{x - 2}$ $\quad$ D: $[-3, \infty)$ $\quad$ R: $[-5, \infty)$

4. Find $f(g(x))$ and $g(f(x))$ and their domain and range if $f(x) = \dfrac{x-1}{x}$ and $g(x) = \dfrac{2x+1}{2-x}$

$$f(g(x)) = \frac{\frac{2x+1}{2-x} - 1}{\frac{2x+1}{2-x}} = \frac{\frac{2x+1}{2-x} - \frac{2-x}{2-x}}{\frac{2x+1}{2-x}} = \boxed{\frac{3x-1}{2x+1}}$$

D: $\left(-\infty, \dfrac{-1}{2}\right) \cup \left(\dfrac{-1}{2}, 2\right) \cup (2, \infty)$

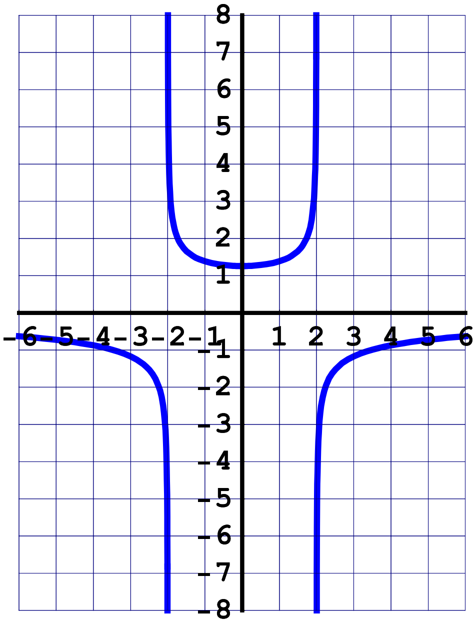
R: $(-\infty, 1) \cup \left(1, \dfrac{3}{2}\right) \cup \left(\dfrac{3}{2}, \infty\right)$

$$g(f(x)) = \frac{2\left(\frac{x-1}{x}\right) + 1}{2 - \frac{x-1}{x}} = \frac{2\left(\frac{x-1}{x}\right) + \frac{x}{x}}{\frac{2x}{x} - \frac{x-1}{x}} = \boxed{\frac{3x-2}{x+1}}$$

D: $(-\infty, -1) \cup (-1, 0) \cup (0, \infty)$

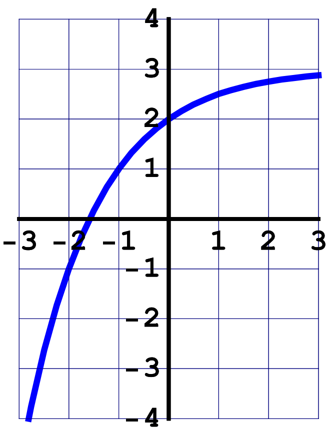
R: $(-\infty, -2) \cup (-2, 3) \cup (3, \infty)$

5. Use your calculator to help you determine the domain and range of the function $y = \dfrac{2}{\sqrt[3]{4 - x^2}}$

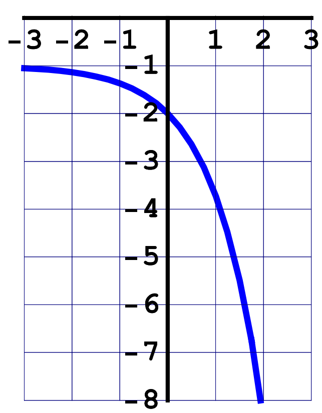


D: $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$ **R:** $(-\infty, 0) \cup \left(\sqrt[3]{2}, \infty\right)$

6. Graph $y = -\left(\dfrac{1}{2}\right)^x + 3$



7. Graph $y = -e^x - 1$



8. Change the bases: (a) 27^{2x}, base 3 (b) $\left(\frac{1}{16}\right)^{\frac{x}{2}}$, base 2

(a) $27^{2x} = (3^3)^{2x} = \boxed{3^{6x}}$ (b) $\left(\frac{1}{16}\right)^{\frac{x}{2}} = (2^{-4})^{\frac{x}{2}} = \boxed{2^{-2x}}$

9. Solve $e^{-x} - 4 = 0$

$e^{-x} = 4 \rightarrow -x = \ln 4 \rightarrow \boxed{x = -\ln 4}$

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10. The population of Cupertino was 800 in the year 1880. Assume that the population increased at the rate of 4% a year (a) Estimate the population in 1920 (b) When did the population reach 10, 000?

(a) $y = 800(1.04)^x \rightarrow 800(1.04)^{40} = 3841$ people

(b) $10,000 = 800(1.04)^x \rightarrow \frac{25}{2} = (1.04)^x \rightarrow \ln\left(\frac{25}{2}\right) = x\ln(1.04) \rightarrow x = \frac{\ln 25 - \ln 2}{\ln 26 - \ln 25} \approx 64.398$ years

so the population reached 10, 000 people in $\boxed{1944.}$

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11. The half – life of the radioactive isotope Calculusium is 8 days. There are 36 grams present initially. When will there be 5 grams remaining?

$y = k\left(\left(\frac{1}{2}\right)^{\frac{t}{\text{life}}}\right) \rightarrow 5 = 36\left(\left(\frac{1}{2}\right)^{\frac{t}{8}}\right) \rightarrow \frac{5}{36} = \left(\frac{1}{2}\right)^{\frac{t}{8}} \rightarrow \ln 5 - \ln 36 = \frac{t}{8}(\ln 1 - \ln 2)$

$\rightarrow t = 8\frac{(\ln 36 - \ln 5)}{\ln 2}$ days or $\boxed{t \approx 22.784 \text{ days}}$

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12. Determine how much time is required to double your investment if interest is earned at the rate of 7.25% compounded continuously.

$2A = Ae^{0.0725t} \rightarrow 2 = e^{0.0725t} \rightarrow \ln 2 = 0.0725t \rightarrow t = \frac{\ln 2}{0.0725} =$ years

or $\frac{400\ln 2}{29}$ years or $\boxed{t \approx 9.561 \text{ years}}$