1.4 and 1.5 Parametric Equations and Inverse Functions

Parametric Equations and Parametric Curves

If x and y are given as functions $x = f(t)$, and $y = g(t)$, over an interval of t – values, then the set of points $(x, y) = (f(t), g(t))$ defined by these equations is a parametric curve. The equations are parametric equations for the curve.

One – to – One Functions

A function $f(x)$ is one – to – one on a domain D if $f(a) \neq f(b)$ whenever $a \neq b$. You may check this graphically with a horizontal line test.

Inverse Functions

Let $f(x)$ be a one – to – one function with domain D and range R. The function $f^{-1}(x)$ is the inverse of $f(x)$ if and only if :

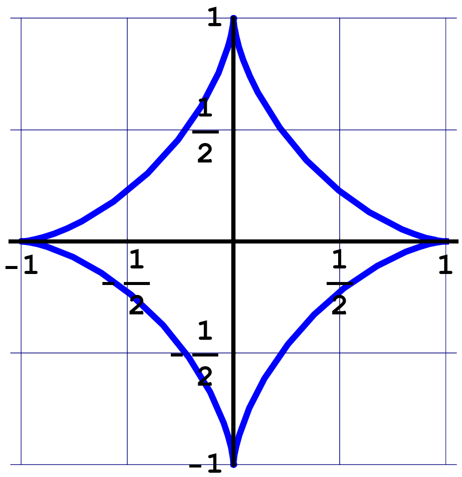
(a) $f^{-1}(x)$ has domain R and range D

(b) $f^{-1}(f(x)) = x$ for every x in D

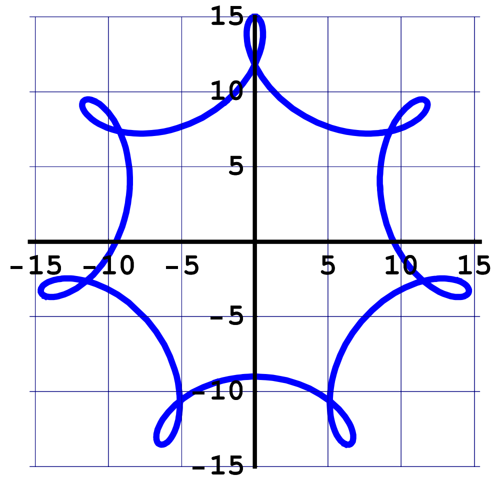
(c) $f(f^{-1}(y)) = y$ for every y in R

Let's try some problems from the book . . .

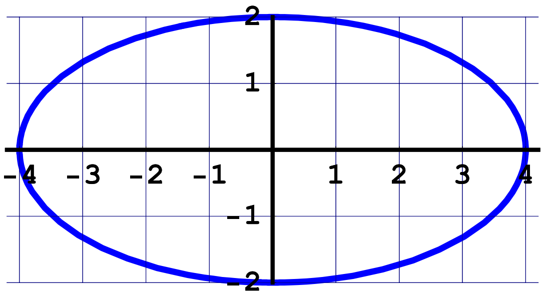
1. (2) $x = \sin^3 t$, $y = \cos^3 t$



2. (4) $x = 12\sin t - 3\sin(6t)$, $y = 12\cos t + 3\cos(6t)$



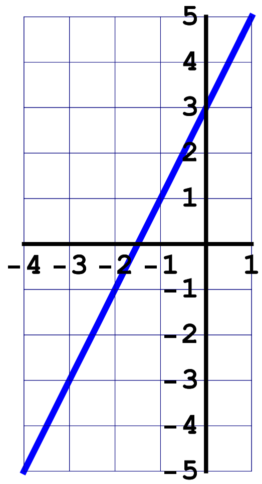
3. (10) $x = 4\cos t, \quad y = 2\sin t, \quad 0 \le t \le 2\pi$



$\cos^2 t + \sin^2 t = 1 \qquad \cos t = \frac{x}{4} \qquad \sin t = \frac{y}{2}$

$\rightarrow \boxed{\frac{x^2}{16} + \frac{y^2}{4} = 1}$

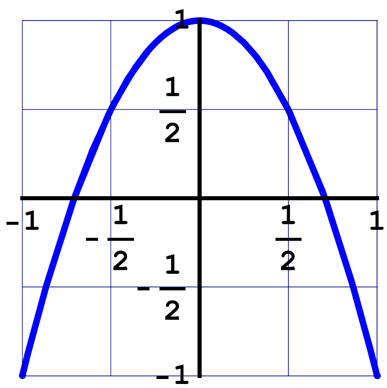
4. (19) $x = 2t - 5, \quad y = 4t - 7, \quad -\infty < t < \infty$



$t = \frac{x+5}{2} \qquad y = 4\left(\frac{x+5}{2}\right) - 7 \qquad \rightarrow \boxed{y = 2x + 3}$

$\boxed{\text{D: All Reals} \quad \text{R: All Reals}}$

5. (25) $x = \sin t, \quad y = \cos(2t), \quad -\infty < t < \infty$



$\cos(2t) = 1 - 2\sin^2 t \qquad \rightarrow \boxed{y = 1 - 2x^2 \qquad \text{D: } [-1, 1] \qquad \text{R: } [-1, 1]}$

6. Find a parametrization for the line segment with endpoints $(-2, 3)$ and $(4, -5)$.
Begin with the first point, using it to set up the following functions for x and y

$x = -2 + at \qquad y = 3 + bt \qquad \rightarrow \qquad t = 2 \qquad 4 = -2 + 2a \qquad -5 = 3 + 2b$

$a = 3 \qquad b = -4 \qquad \boxed{x = -2 + 3t \qquad y = 3 - 4t \qquad t \text{ in } [0, 2]}$

For problems 7 – 10, determine whether the function has an inverse.

7. $f(x) = x^2$

an even function
so not one – to – one
and **has no inverse**

8. $f(x) = \cos x$

an even function
so not one – to – one
and **has no inverse**

9. $f(x) = e^{x^2}$

an even function
so not one – to – one
and **has no inverse**

10. $f(x) = \dfrac{3x}{2x+1}$

graph the function on your
calculator → it is one – to – one
so it **has an inverse**

For problems 11 and 12, find f^{-1}.

11. $f(x) = 3x - 1$

$y = 3x - 1 \quad \rightarrow \quad x = 3y - 1$

$\rightarrow \quad 3y = x + 1 \quad \rightarrow \quad \boxed{y = \dfrac{x+1}{3}}$

12. $f(x) = \dfrac{1-2x}{x+4}$

$x = \dfrac{1-2y}{y+4} \qquad xy + 4x = 1 - 2y \qquad$ so

$2y + xy = 1 - 4x \quad \rightarrow \quad \boxed{y = \dfrac{1-4x}{2+x}}$

13. Use parametric graphing to graph f, f^{-1}, and $y = x$ if $f(x) = \log_3 x$