1.4 and 1.5 Parametric Equations and Inverse Functions

Parametric Equations and Parametric Curves

If x and y are given as functions $x = f(t)$, and $y = g(t)$, over an interval of t – values, then the set of points $(x, y) = (f(t), g(t))$ defined by these equations is a parametric curve. The equations are parametric equations for the curve.

One – to – One Functions

A function $f(x)$ is one – to – one on a domain D if $f(a) \neq f(b)$ whenever $a \neq b$. You may check this graphically with a horizontal line test.

Inverse Functions

Let $f(x)$ be a one – to – one function with domain D and range R. The function $f^{-1}(x)$ is the inverse of $f(x)$ if and only if :

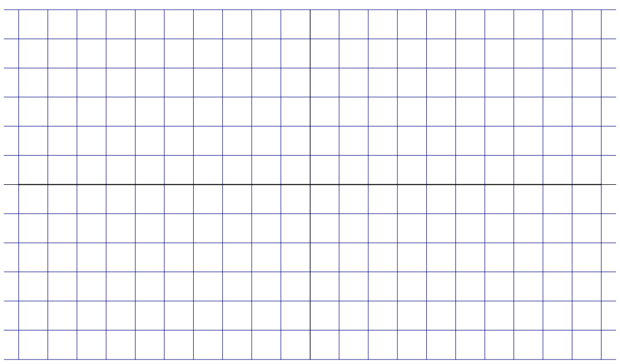
(a) $f^{-1}(x)$ has domain R and range D

(b) $f^{-1}(f(x)) = x$ for every x in D

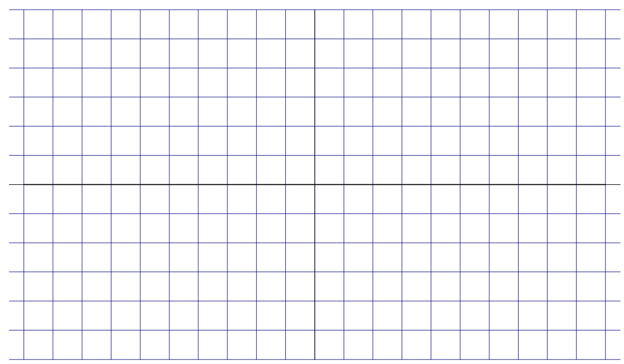
(c) $f(f^{-1}(y)) = y$ for every y in R

Let's try some problems from the book . . .

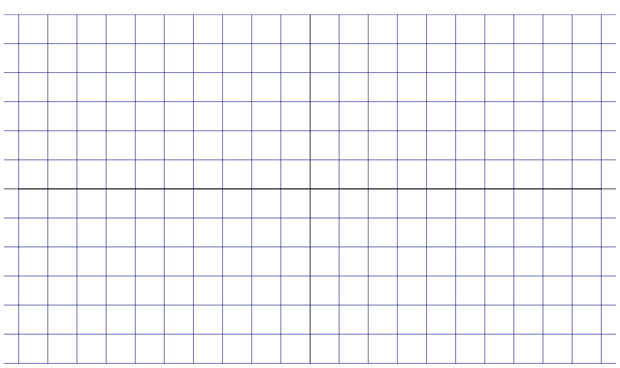
1. (2) $x = \sin^3 t, \quad y = \cos^3 t$



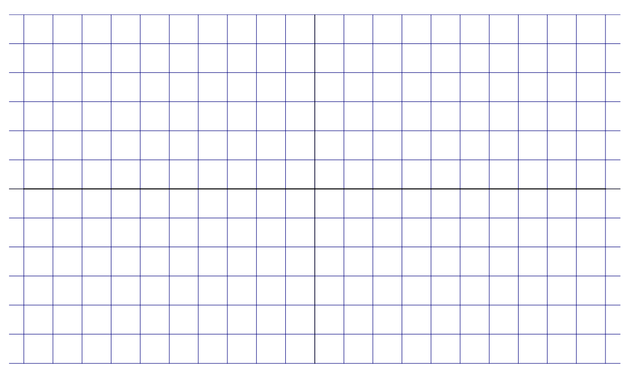
2. (4) $x = 12 \sin t - 3 \sin(6t), \quad y = 12 \cos t + 3 \cos(6t)$



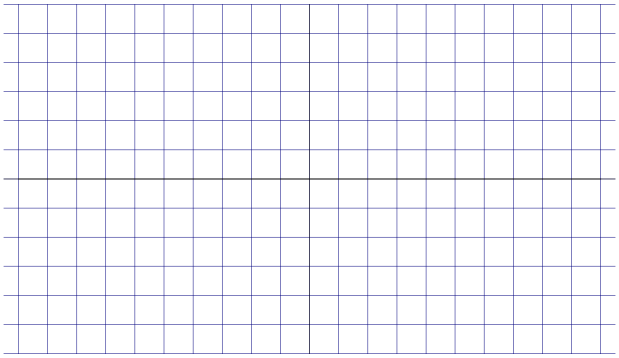
3. (10) $x = 4 \cos t, \quad y = 2 \sin t, \quad 0 \leq t \leq 2\pi$



4. (19) $x = 2t - 5, \quad y = 4t - 7, \quad -\infty < t < \infty$



5. (25) $x = \sin t, \quad y = \cos(2t), \quad -\infty < t < \infty$



6. Find a parametrization for the line segment with endpoints $(-2, 3)$ and $(4, -5)$.

For problems 7 – 10, determine whether the function has an inverse.

7. $f(x) = x^2$

8. $f(x) = \cos x$

9. $f(x) = e^{x^2}$

10. $f(x) = \dfrac{3x}{2x + 1}$

For problems 11 and 12, find f^{-1}.

11. $f(x) = 3x - 1$

12. $f(x) = \dfrac{1 - 2x}{x + 4}$

13. Use parametric graphing to graph f, f^{-1}, and $y = x$ if $f(x) = \log_3 x$