

# 1.5 and 1.6 Logarithmic and Trigonometric Functions

## Relationship between Exponential and Logarithmic Functions

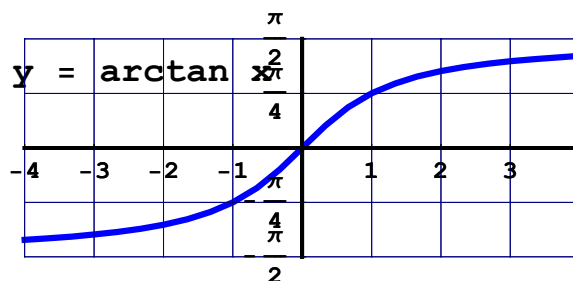
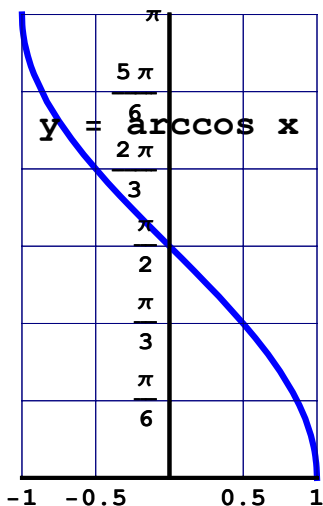
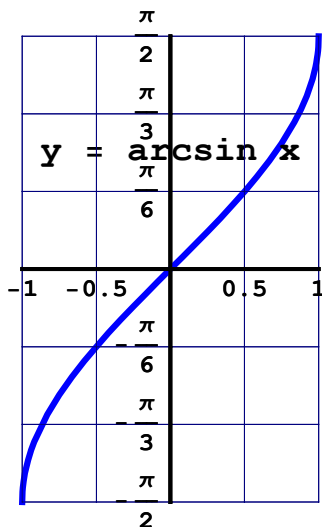
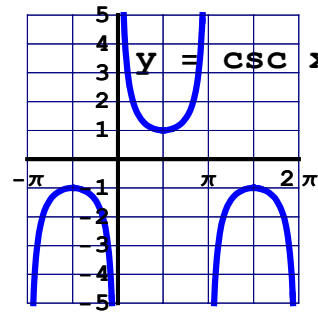
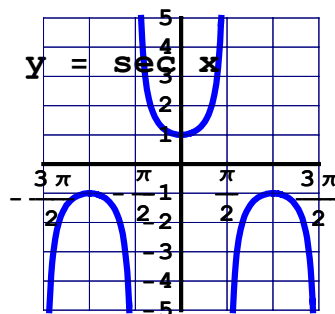
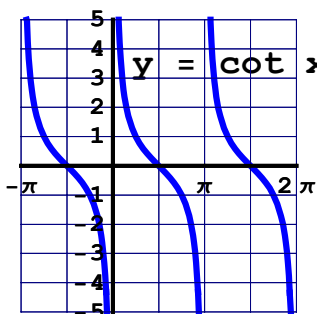
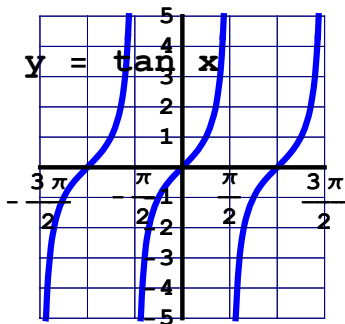
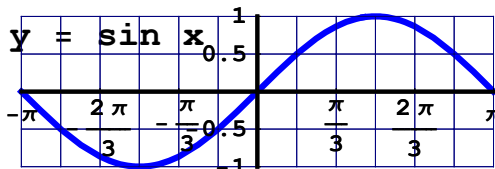
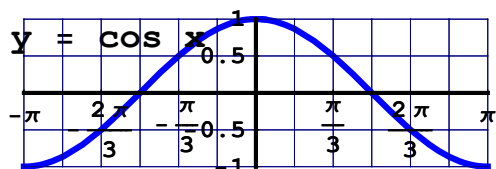
- (1)  $y = \log_a x$  is the inverse of  $y = a^x$ , where  $a > 0$ ,  $a \neq 1$   
 (2)  $a^{\log_a x} = x$  and  $\log_a (a^x) = x$ , where  $a > 1$ ,  $x > 0$

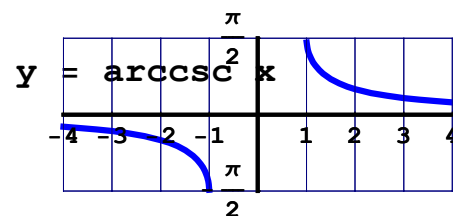
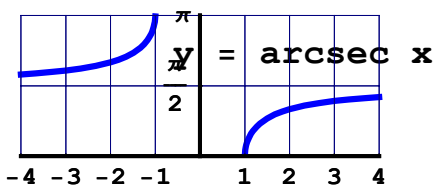
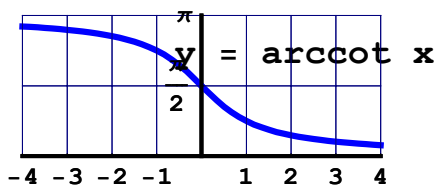
## Rules for Logarithms

Assuming that  $x > 0$  and  $y > 0$ ,

- (1)  $\log_a (xy) = \log_a x + \log_a y$       (2)  $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$       (3)  $\log_a (x^y) = y \log_a x$   
 (4)  $\log_a b = \frac{\log_c b}{\log_c a}$  (where  $a > 0$ ,  $b > 0$ ,  $c > 0$ ,  $a \neq 1$ ,  $b \neq 1$ ,  $c \neq 1$ )

## Trigonometric Functions and their Inverses

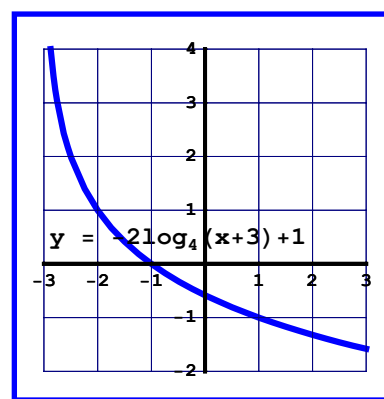
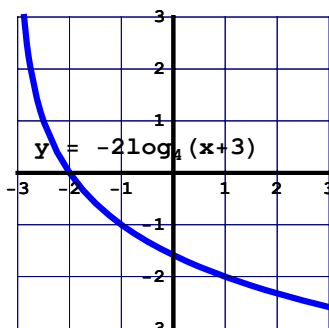
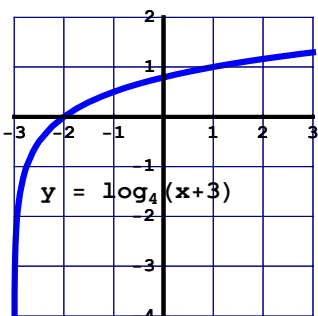
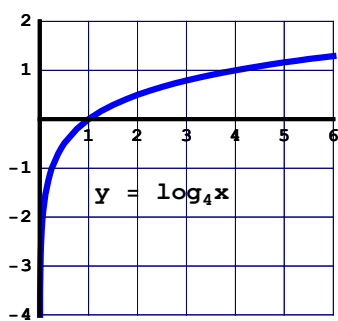




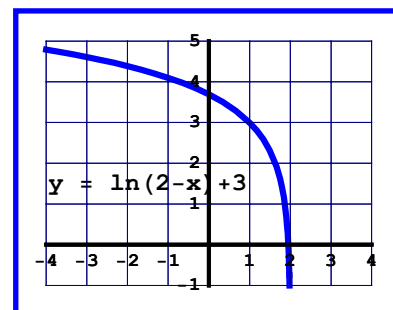
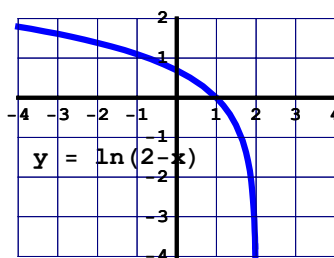
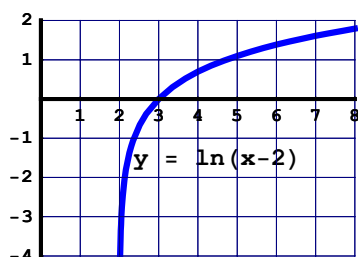
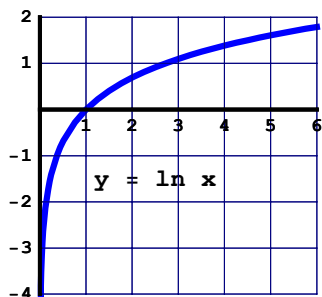
### Arc Length for a Circle

$$L = \theta r$$

1. Graph the function  $y = -2 \log_4(x+3) - 1$



2. Graph the function  $y = \ln(2-x) + 3$



3. Solve the equation  $2^{3x} - 5 = 0$

$$2^{3x} = 5 \rightarrow 3x = \log_2 5 \rightarrow x = \frac{1}{3} \log_2 5$$

4. Solve the equation  $3^x - 5 = -6(3^{-x})$

Multiply both sides by  $3^x$ , to get

$$(3^x)^2 - 5(3^x) + 6 = 0 \rightarrow (3^x - 2)(3^x - 3) = 0$$

$$x = \log_3 2, 1$$

5. Solve for  $y$  if  $\ln(x+2) - 3 = \ln 4 - \ln(y+1)$

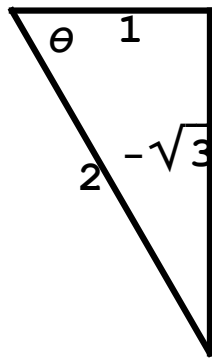
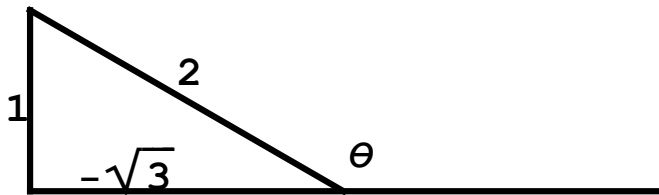
$$\ln(y+1) - \ln 4 + \ln(x+2) = 3 \rightarrow \ln\left(\frac{(y+1)(x+2)}{4}\right) = 3 \rightarrow \frac{(y+1)(x+2)}{4} = e^3 \text{ so}$$

$$y = \frac{4e^3}{x+2} - 1$$

For problems 6 and 7, find the measure of the angle in radians and degrees

6.  $\cot^{-1}(-\sqrt{3})$

7.  $\csc^{-1}\left(\frac{-2}{\sqrt{3}}\right)$

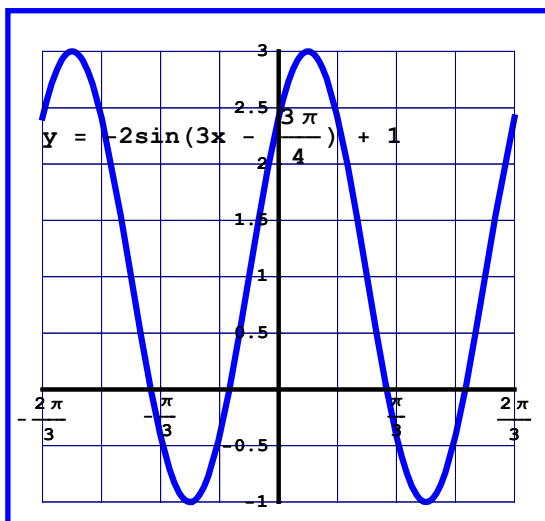
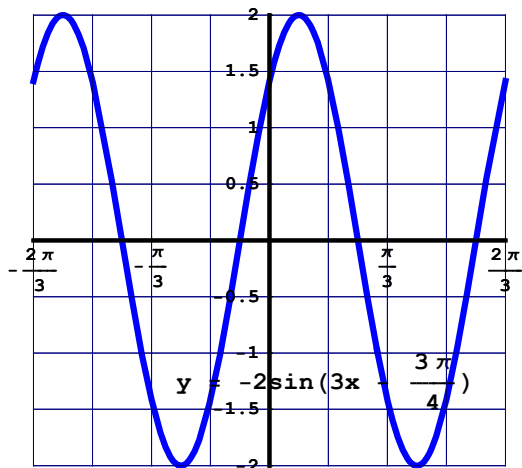
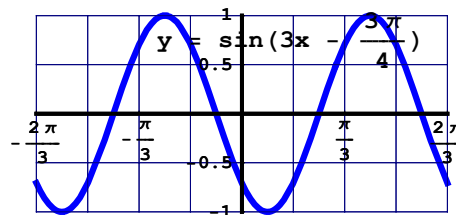
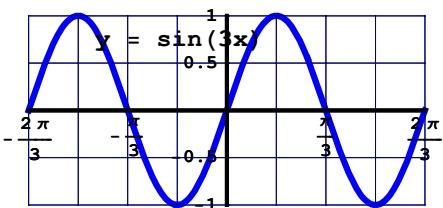
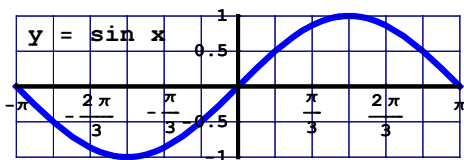


$\theta = \frac{5\pi}{6}, 150^\circ$

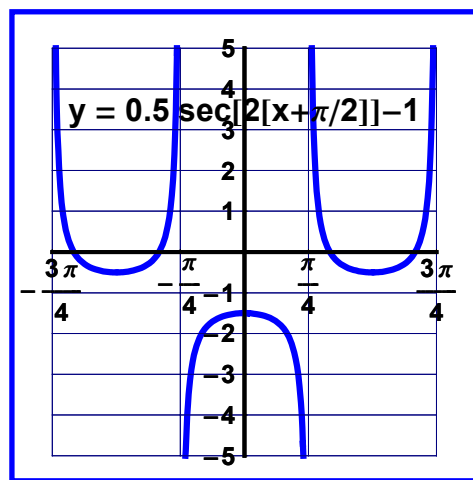
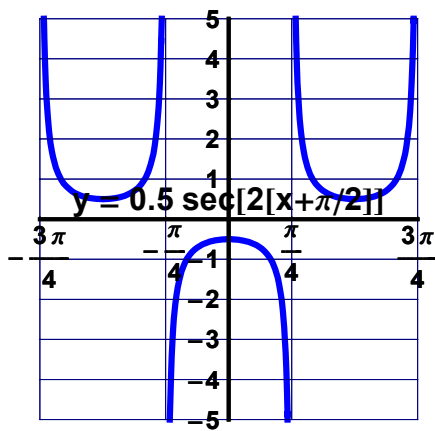
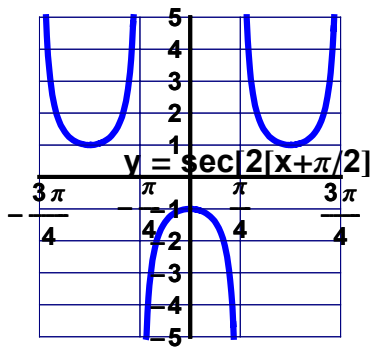
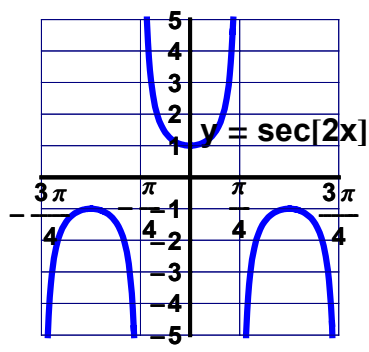
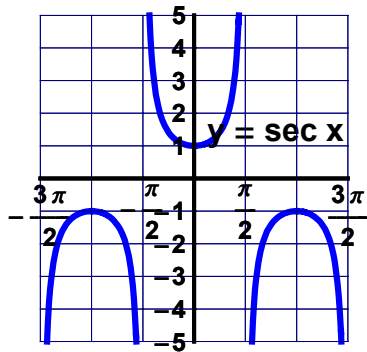
$\frac{-\pi}{3}, -60^\circ$

For problems 8–10, graph the function, and find the (a) period, and (b) amplitude

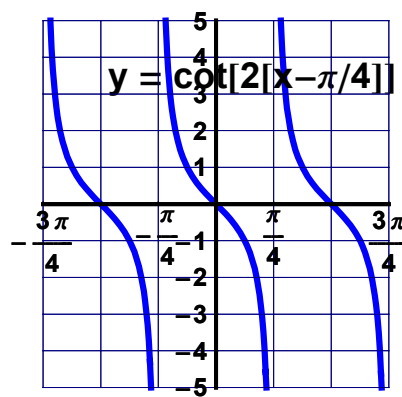
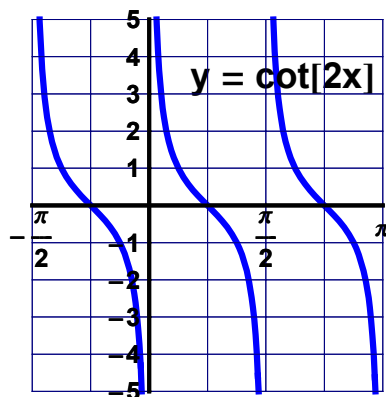
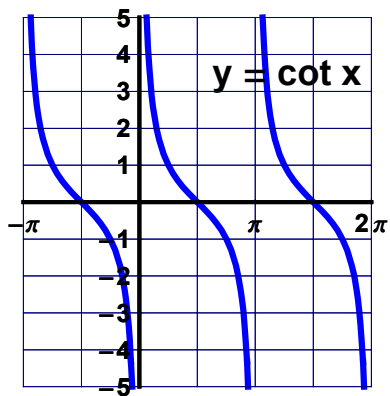
8.  $y = -2\sin\left(3x - \frac{3\pi}{4}\right) + 1$

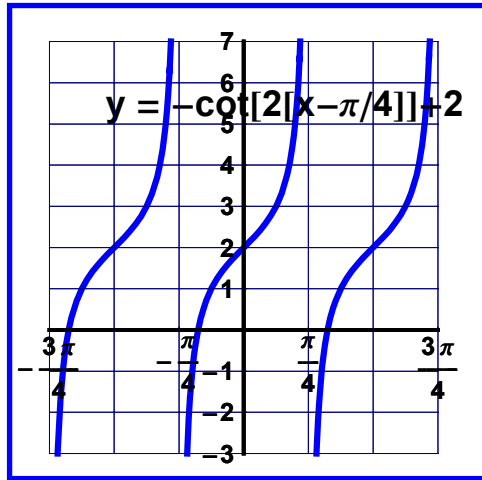
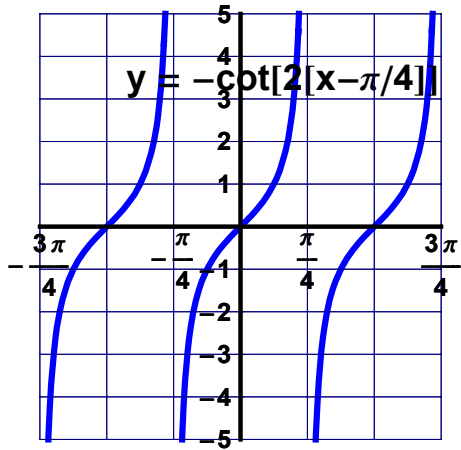


$$9. y = \frac{1}{2} \sec(2x + \pi) - 1$$

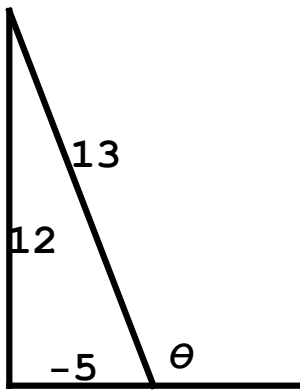


$$10. y = -\cot\left(2x - \frac{\pi}{2}\right) + 2$$





11. Find the values of the six trigonometric functions if  $\theta = \sec^{-1}\left(\frac{-13}{5}\right)$



$$\cos \theta = \frac{-5}{13}, \quad \sin \theta = \frac{12}{13}, \quad \tan \theta = \frac{-12}{5}, \quad \sec \theta = \frac{-13}{5}, \quad \csc \theta = \frac{13}{12}, \quad \cot \theta = \frac{-5}{12}$$