2.1 Rate of Change and Limits

One – sided and Two – sided Limits

$\lim_{x \to c} f(x) = L$ if and only if $\lim_{x \to c^-} f(x) = L$ and $\lim_{x \to c^+} f(x) = L$

The Sandwich Theorem

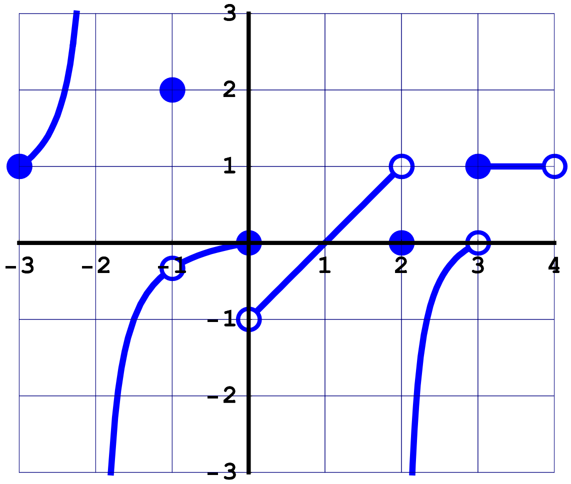
If $g(x) \le f(x) \le h(x)$ for all $x \ne c$ in some interval about c, and

$\lim_{x \to c} g(x) = \lim_{x \to c} h(x) = L$, then $\lim_{x \to c} f(x) = L$

An Important Limit

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

1. For any $\lim_{x \to c} f(x)$, we are not looking for what happens at $x = c$, but at what happens as x approaches c.



$\lim_{x \to -2^-} f(x) = \infty$	$\lim_{x \to -2^+} f(x) = -\infty$	$\lim_{x \to -2} f(x) =$ DNE
$\lim_{x \to -1^-} f(x) = \frac{-1}{3}$	$\lim_{x \to -1^+} f(x) = \frac{-1}{3}$	$\lim_{x \to -1} f(x) = \frac{-1}{3}$
$\lim_{x \to 0^-} f(x) = 0$	$\lim_{x \to 0^+} f(x) = -1$	$\lim_{x \to -2} f(x) =$ DNE
$\lim_{x \to 1^-} f(x) = 0$	$\lim_{x \to 1^+} f(x) = 0$	$\lim_{x \to 1} f(x) = 0$
$\lim_{x \to 2^-} f(x) = 1$	$\lim_{x \to 2^+} f(x) = -\infty$	$\lim_{x \to 2} f(x) =$ DNE
$\lim_{x \to 3^-} f(x) = 0$	$\lim_{x \to 3^+} f(x) = 1$	$\lim_{x \to 3} f(x) =$ DNE

For problems 2 – 7, find the limit, if it exists.

2. $\lim_{x \to 3} \frac{x+3}{x-1}$

$= \frac{3+3}{3-1} = \boxed{3}$

3. $\lim_{x \to -2} \pi$

$= \boxed{\pi}$

4. $\lim_{x \to -3} \dfrac{x^2 - x - 12}{x^2 + 4x + 3}$

$= \lim_{x \to -3} \dfrac{(x - 4)(x + 3)}{(x + 1)(x + 3)} = \lim_{x \to -3} \dfrac{(x - 4)}{(x + 1)} = \boxed{\dfrac{7}{2}}$

5. $\lim_{x \to 25} \dfrac{\sqrt{x} - 5}{x - 25}$

$= \lim_{x \to 25} \dfrac{\sqrt{x} - 5}{(\sqrt{x} - 5)(\sqrt{x} + 5)} = \lim_{x \to 25} \dfrac{1}{(\sqrt{x} + 5)} = \boxed{\dfrac{1}{10}}$

6. $\lim_{h \to 0} \dfrac{(x + h)^2 - x^2}{h}$

$= \lim_{h \to 0} \dfrac{x^2 + 2xh + h^2 - x^2}{h} = \lim_{h \to 0} \dfrac{2xh + h^2}{h}$

$= \lim_{h \to 0} (2x + h) = \boxed{2x}$

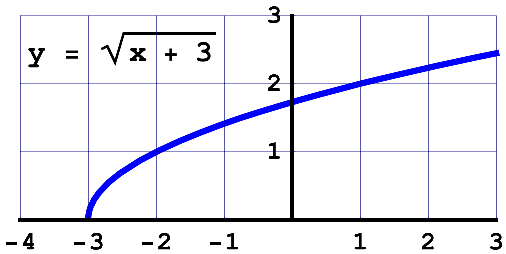
7. $\lim_{x \to 0} \dfrac{\sin(2x)}{2\sin(3x)}$

$= \lim_{x \to 0} \dfrac{\sin(2x)}{2x} \left(\dfrac{3x}{\sin(3x)}\right)\left(\dfrac{1}{3}\right) = \boxed{\dfrac{1}{3}}$

For problems 8 – 12, find (a) $\lim_{x \to c^-} f(x)$, (b) $\lim_{x \to c^+} f(x)$, and (c) $\lim_{x \to c} f(x)$, if they exist.

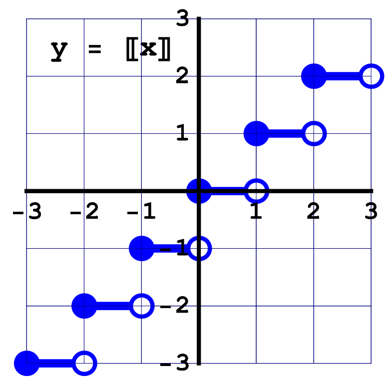
8. $c = -3$, $f(x) = \sqrt{x + 3}$

$\lim_{x \to -3^-} f(x) = \text{DNE}$ $\quad \lim_{x \to -3^+} f(x) = 0$ $\quad \lim_{x \to -3} f(x) = \text{DNE}$



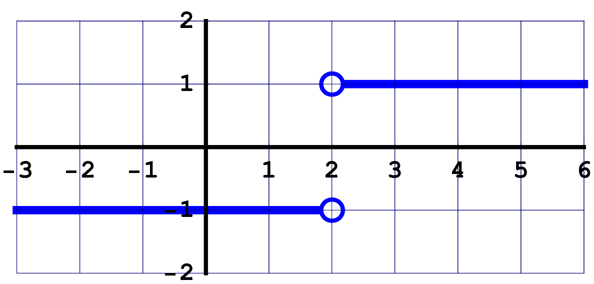
9. $c = -1$, $f(x) = [\![x]\!]$ (or $f(x) = \text{int } x$)

$\lim_{x \to -1^-} f(x) = -2$ $\quad \lim_{x \to -1^+} f(x) = -1$ $\quad \lim_{x \to -1} f(x) = \text{DNE}$



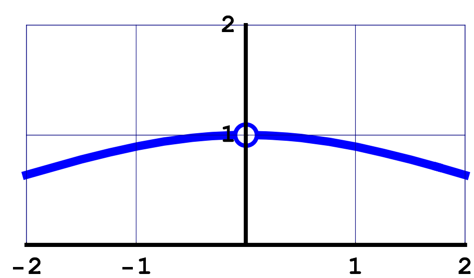
10. $c = 2$, $f(x) = \dfrac{|x - 2|}{x - 2}$

$\lim_{x \to 2^-} f(x) = -1$ $\quad \lim_{x \to 2^+} f(x) = 1$ $\quad \lim_{x \to 2} f(x) = \text{DNE}$



11. $c = 0$, $f(x) = \dfrac{x + 2\sin x}{3x}$

$\lim_{x \to 1^-} f(x) = 1$ $\quad \lim_{x \to 1^+} f(x) = 1$ $\quad \lim_{x \to 1} f(x) = 1$



12. $c = 3$, $f(x) = \begin{cases} x^2 - 4, & x < 3 \\ 4, & x = 3 \\ 5 - x, & x > 3 \end{cases}$

$\lim_{x \to 3^-} f(x) = 5$ $\lim_{x \to 3^+} f(x) = 2$ $\lim_{x \to 3} f(x) = \text{DNE}$