2.1 Rate of Change and Limits

One – sided and Two – sided Limits

$\lim_{x \to c} f(x) = L$ if and only if $\lim_{x \to c^-} f(x) = L$ and $\lim_{x \to c^+} f(x) = L$

The Sandwich Theorem

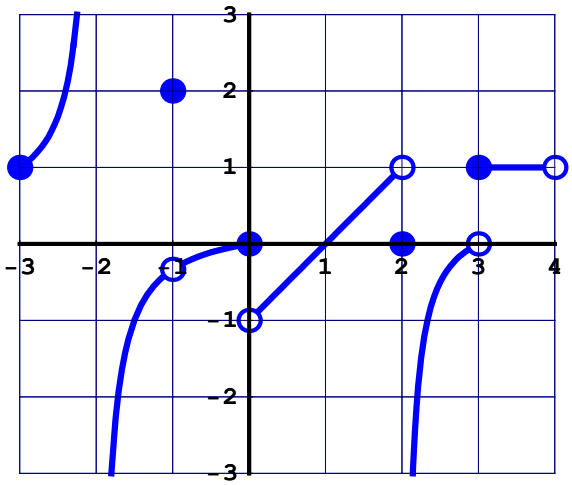
If $g(x) \le f(x) \le h(x)$ for all $x \neq c$ in some interval about c, and

$\lim_{x \to c} g(x) = \lim_{x \to c} h(x) = L$, then $\lim_{x \to c} f(x) = L$

An Important Limit

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

1. For any $\lim_{x \to c} f(x)$, we are not looking for what happens at $x = c$, but at what happens as x approaches c.



For problems 2 – 7, find the limit, if it exists.

2. $\lim_{x \to 3} \frac{x + 3}{x - 1}$

3. $\lim_{x \to -2} \pi$

4. $\lim_{x \to -3} \frac{x^2 - x - 12}{x^2 + 4x + 3}$

5. $\lim_{x \to 25} \frac{\sqrt{x} - 5}{x - 25}$

6. $\lim_{h \to 0} \dfrac{(x + h)^2 - x^2}{h}$

7. $\lim_{x \to 0} \dfrac{\sin(2x)}{2\sin(3x)}$

For problems 8 – 12, find (a) $\lim_{x \to c^-} f(x)$, (b) $\lim_{x \to c^+} f(x)$, and (c) $\lim_{x \to c} f(x)$, if they exist.

8. $c = -3$, $f(x) = \sqrt{x + 3}$

9. $c = -1$, $f(x) = [\![x]\!]$ (or $f(x) = \text{int } x$)

10. $c = 2$, $f(x) = \dfrac{|x - 2|}{x - 2}$

11. $c = 0$, $f(x) = \dfrac{x + 2\sin x}{3x}$

12. $c = 3$, $f(x) = \begin{cases} x^2 - 4, & x < 3 \\ 4, & x = 3 \\ 5 - x, & x > 3 \end{cases}$