2.2 Limits Involving Infinity

Horizontal Asymptote

$y = b$ is a horizontal asymptote if either $\lim_{x \to \infty} f(x) = b$ or $\lim_{x \to -\infty} f(x) = b$

Vertical Asymptote

$x = a$ is a vertical asymptote if either $\lim_{x \to a^+} f(x) = \pm\infty$ or $\lim_{x \to a^-} f(x) = \pm\infty$

End Behavior Model

g is

(a) a right end behavior model for f if and only if $\lim_{x \to \infty} \dfrac{f(x)}{g(x)} = 1$

(b) a left end behavior model for f if and only if $\lim_{x \to -\infty} \dfrac{f(x)}{g(x)} = 1$

Inequalities Relating Some Important Functions

As $x \to \infty$, $\log_a x < x^b < c^x$ for all constants $a > 1$, $b > 0$, $c > 1$

For problems 1 – 4, find (a) $\lim_{x \to \infty} f(x)$ (b) $\lim_{x \to -\infty} f(x)$, and

(c) identify all horizontal asymptotes

1. $f(x) = \dfrac{\sqrt{|x|}}{2^{-x}}$

2. $f(x) = \dfrac{4x - 3}{|x - 5|}$

3. $f(x) = \dfrac{x}{\sqrt{x^2 + 1}}$

4. $f(x) = \dfrac{3x + \sin(3x)}{x}$

For problems 5 – 8, find the limits.

5. $\lim_{x \to 0^+} \dfrac{[\![x]\!]}{x}$

6. $\lim_{x \to 0^+} \dfrac{x}{[\![x]\!]}$

7. $\lim_{x \to 0^-} \cot x$

8. $\lim_{x \to -3^-} \dfrac{-2}{x+3}$

For problems 9 – 11, find the vertical asymptotes of the graph of f.

9. $f(x) = \dfrac{3}{x^3 - 8}$

10. $f(x) = \csc x$

11. $f(x) = \dfrac{3 + x}{2x^2 + 5x - 3}$

For problems 12 – 15, find (a) a simple basic function as a right end behavior model, and (b) a simple basic function as a left end behavior model

12. $f(x) = \dfrac{3x^4 - 2x + 1}{x^2 + 10x + 7}$

13. $f(x) = 2^{-x} + x^5$

14. $f(x) = \log_2 |x| + \sqrt[3]{x}$

15. $f(x) = \cos(2x) + x$

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16. $f(2) = 1$, $\lim_{x \to \infty} f(x) = -1$, $\lim_{x \to -\infty} f(x) = 1$, $\lim_{x \to 2^+} f(x) = \infty$, $\lim_{x \to 2^-} f(x) = 0$, $\lim_{x \to -1} f(x) = -\infty$