2.3 Continuity

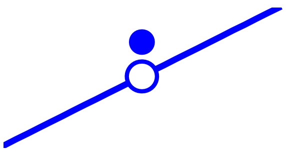
Definition of Continuity at a Point

A. Interior Point: $y = f(x)$ is continuous at an interior point c of its domain if $\lim_{x \to c} f(x) = f(c)$

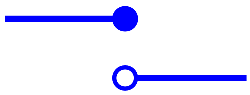
B. Endpoint: $y = f(x)$ is continuous at a left endpoint a or is continuous at a right endpoint b of its domain if $\lim_{x \to a^+} f(x) = f(a)$ or $\lim_{x \to b^-} f(x) = f(b)$ respectively

Types of Discontinuities

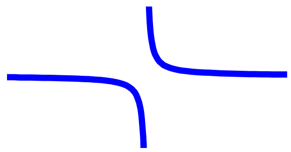
A. Removable



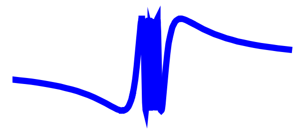
B. Jump



C. Infinite



D. Oscillating

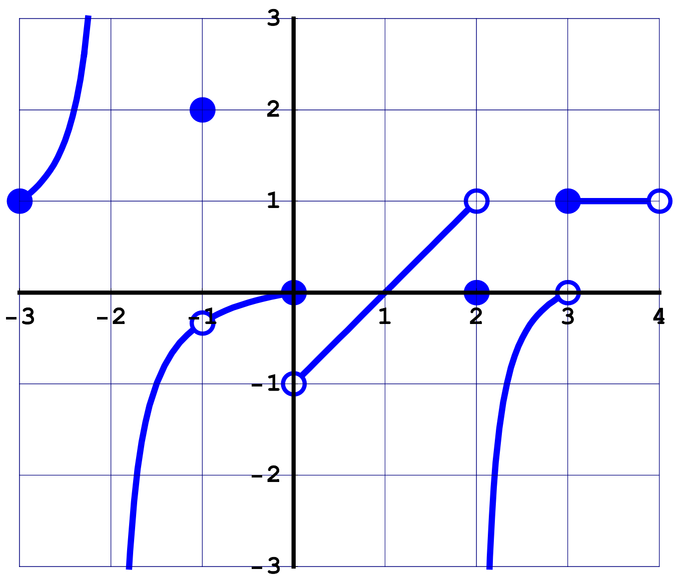


Intermediate Value Theorem

If $y = f(x)$ is continuous on $[a, b]$, then it takes on every value from $f(a)$ to $f(b)$. Or, if y_0 is between $f(a)$ and $f(b)$, then $y_0 = f(c)$ for some c in $[a, b]$.

1. For the function below, find and classify the discontinuities

Infinite discontinuities at $x = -2$ and $x = 2$, Jump discontinuities at $x = 0$ and $x = 3$, and Removable discontinuity at $x = -1$



For problems 2 – 7, find and classify the discontinuities for the given function.

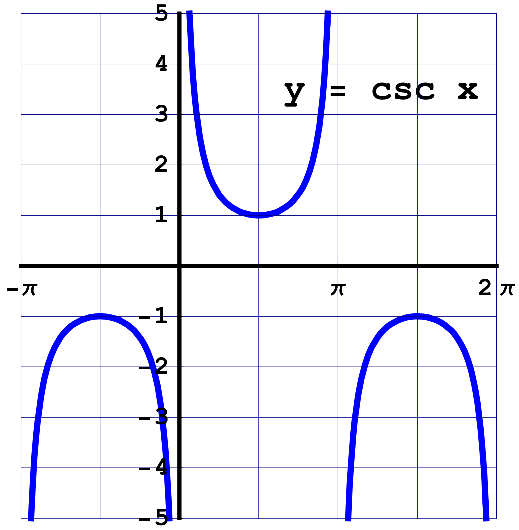
2. $f(x) = \csc x$

for any $x = k\pi$, where k is any integer, the

$\lim_{x \to k\pi^+} \csc x = \infty$ or $-\infty$ and

$\lim_{x \to k\pi^-} \csc x = \infty$ or $-\infty$ so

infinite discontinuities at $x = k\pi$, where k is any integer

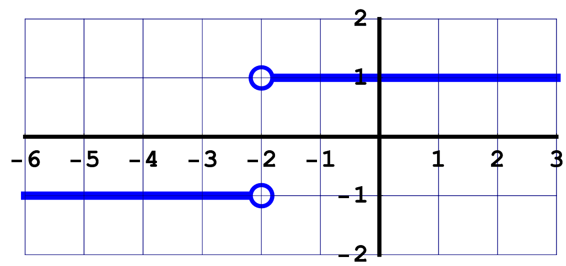


3. $f(x) = \dfrac{|x + 2|}{x + 2}$

consider $x = -2$, $\lim_{x \to -2^+} \dfrac{|x + 2|}{x + 2} = 1$ and

$\lim_{x \to -2^-} \dfrac{|x + 2|}{x + 2} = -1$ so there is

a jump discontinuity at $x = -2$

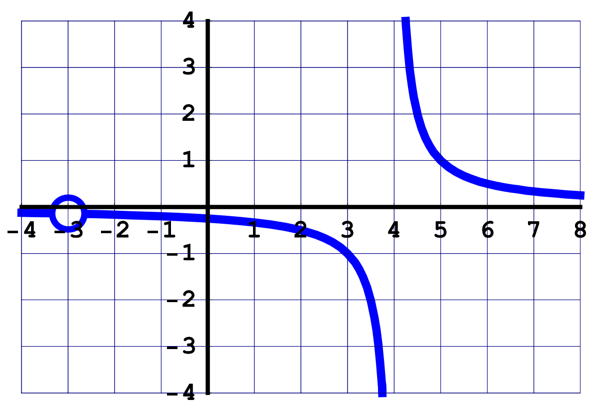


---

4. $f(x) = \dfrac{x + 3}{x^2 - x - 12}$

$f(x) = \dfrac{x + 3}{(x + 3)(x - 4)}$ so

there is a removable discontinuity at $x = -3$, and an infinite discontinuity at $x = 4$

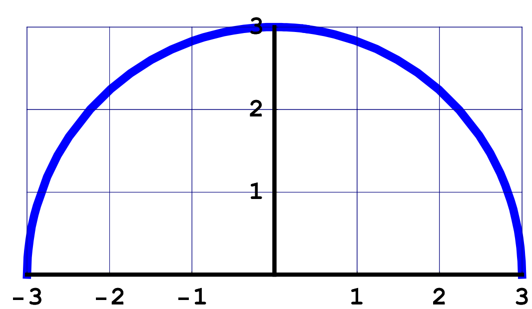


5. $f(x) = \sqrt{9 - x^2}$

this graph of this function is a semicircle, and it is

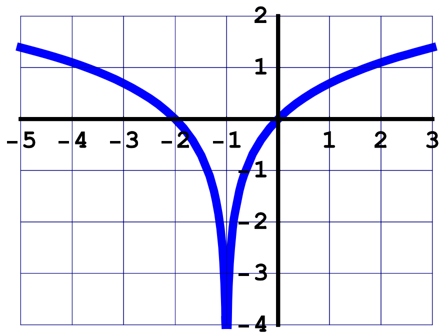
continuous on its domain of $[-3, 3]$ so

there are no discontinuities



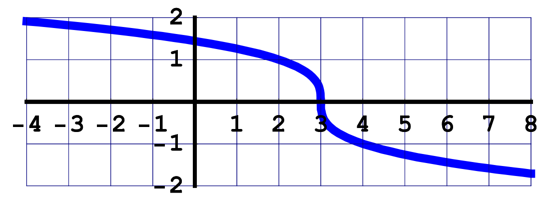
6. $f(x) = \ln|x + 1|$

this function is defined for all reals except $x = -1$, and there is a vertical asymptote at $x = -1$, so there is **an infinite discontinuity at $x = -1$**



7. $f(x) = \sqrt[3]{3 - x}$

the domain is the set of all reals, and there are **no discontinuities**



For problems 8 and 9, give a formula for the extended function that is continuous at the indicated point.

8. $f(x) = \dfrac{\sin\left(\frac{x}{2}\right)}{x}, \quad x = 0$

$$\lim_{x \to 0} \frac{\sin\left(\frac{x}{2}\right)}{x} = \lim_{x \to 0} \frac{\sin\left(\frac{x}{2}\right)}{\frac{x}{2}}\left(\frac{1}{2}\right) = 1\left(\frac{1}{2}\right) = \frac{1}{2}$$

so to "fix" the removable discontinuity, we have

$$f(x) = \begin{cases} \dfrac{\sin\left(\frac{x}{2}\right)}{x}, & x \neq 0 \\ \dfrac{1}{2}, & x = 0 \end{cases}$$

9. $f(x) = \dfrac{x^3 + 1}{x^2 - 1}, \quad x = -1$

$$f(x) = \frac{(x + 1)(x^2 - x + 1)}{(x - 1)(x + 1)} \rightarrow \frac{x^2 - x + 1}{x - 1}$$

so $$\lim_{x \to -1} \frac{x^2 - x + 1}{x - 1} = \frac{3}{-2} = \frac{-3}{2}$$

so to "fix" the removable discontinuity, we have

$$f(x) = \begin{cases} \dfrac{x^3 + 1}{x^2 - 1}, & x \neq -1 \\ \dfrac{-3}{2}, & x = -1 \end{cases}$$

or

$$f(x) = \frac{x^2 - x + 1}{x - 1}$$