2.3 Continuity

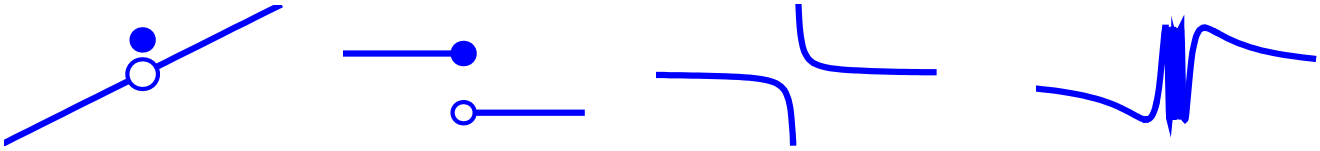
Definition of Continuity at a Point

A. Interior Point: $y = f(x)$ is continuous at an interior point c of its domain if $\lim_{x \to c} f(x) = f(c)$

B. Endpoint: $y = f(x)$ is continuous at a left endpoint a or is continuous at a right endpoint b of its domain if $\lim_{x \to a^+} f(x) = f(a)$ or $\lim_{x \to b^-} f(x) = f(b)$ respectively

Types of Discontinuities

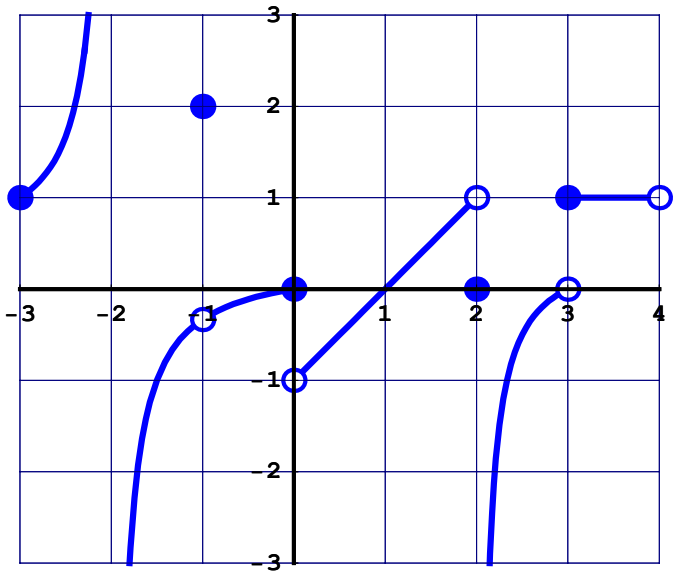
A. Removable **B. Jump** **C. Infinite** **D. Oscillating**



Intermediate Value Theorem

If $y = f(x)$ is continuous on $[a, b]$, then it takes on every value from $f(a)$ to $f(b)$. Or, if y_0 is between $f(a)$ and $f(b)$, then $y_0 = f(c)$ for some c in $[a, b]$.

1. For the function below, find and classify the discontinuities.



For problems 2 – 7, find and classify the discontinuities for the given function.

2. $f(x) = \csc x$

3. $f(x) = \dfrac{|x + 2|}{x + 2}$

4. $f(x) = \dfrac{x + 3}{x^2 - x - 12}$

5. $f(x) = \sqrt{9 - x^2}$

6. $f(x) = \ln|x + 1|$

7. $f(x) = \sqrt[3]{3 - x}$

For problems 8 and 9, give a formula for the extended function that is continuous at the indicated point.

8. $f(x) = \dfrac{\sin\left(\frac{x}{2}\right)}{x}, \quad x = 0$

9. $f(x) = \dfrac{x^3 + 1}{x^2 - 1}, \quad x = -1$