2.4 Rates of Change and Tangent Lines

Slope of a Curve at a Point

The slope of the curve $y = f(x)$ at the point $(a, f(a))$ is the number $m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$, provided that the limit exists.

Normal Line

The normal line to a curve at a point is the line perpendicular to the tangent at that point.

Vertical Tangent

The curve $y = f(x)$ has a vertical tangent at $x = a$ if $\lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \infty$ or if $\lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = -\infty$

For problems 1 and 2, find the average rate of change of the function over each interval.

1. $f(x) = x^2 - 3x$ (a) $[-1, 2]$ (b) $[0, 4]$

2. $f(x) = \sin\left(\frac{x}{3}\right)$ (a) $[-2\pi, \pi]$ (b) $\left[\frac{-\pi}{2}, \frac{3\pi}{4}\right]$

For problems 3 – 6, for the indicated point find (a) the slope of the curve (b) the equation of the tangent line (c) the equation of the normal line

3. $f(x) = x^2 + x$ at $x = 1$

4. $f(x) = |x + 3|$ at $x = -1$

5. $f(x) = \dfrac{2}{x+1}$ at $x = -2$

6. $f(x) = x^3$ at $x = \dfrac{1}{2}$

For problems 7 and 8, find the slope of the curve at $x = a$.

7. $f(x) = x^2 + x$

8. $f(x) = \dfrac{2}{x+1}$

For problems 9 and 10, determine whether the curve has a tangent at the indicated point.

9. $f(x) = \begin{cases} 2 - x^2, & x \le 1 \\ 2 - x, & x > 1 \end{cases}$ at $x = 1$

10. $f(x) = \begin{cases} \dfrac{1}{2}x^2 + x, & x < 2 \\ 3x - 2, & x \ge 2 \end{cases}$ at $x = 2$

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11. Find the rate of change of the surface area of a cube with respect to the length of a side, when the length of the side is $s = 3$ inches.

12. A TI – 89 is dropped from the second floor of the D building. If the height of the calculator is described by $h(t) = 6 - 4.9t^2$ meters, find the velocity of the calculator 1 second after it is dropped.

13. At what point is the tangent to $f(x) = 4 + 6x + x^2$ horizontal?

14. Find the equations of all tangent lines that pass through the point $(1, 1)$ for the function $f(x) = 1 - \frac{1}{2}x^2$