3.1 Derivative of a Function

Derivative

The derivative of the function f with respect to the variable x is the function f′, whose value at x is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}, \quad \text{provided the limit exists.}$$

Derivative (Alternate Definition)

The derivative of the function f at the point x = a is the limit $f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$, provided the limit exists.

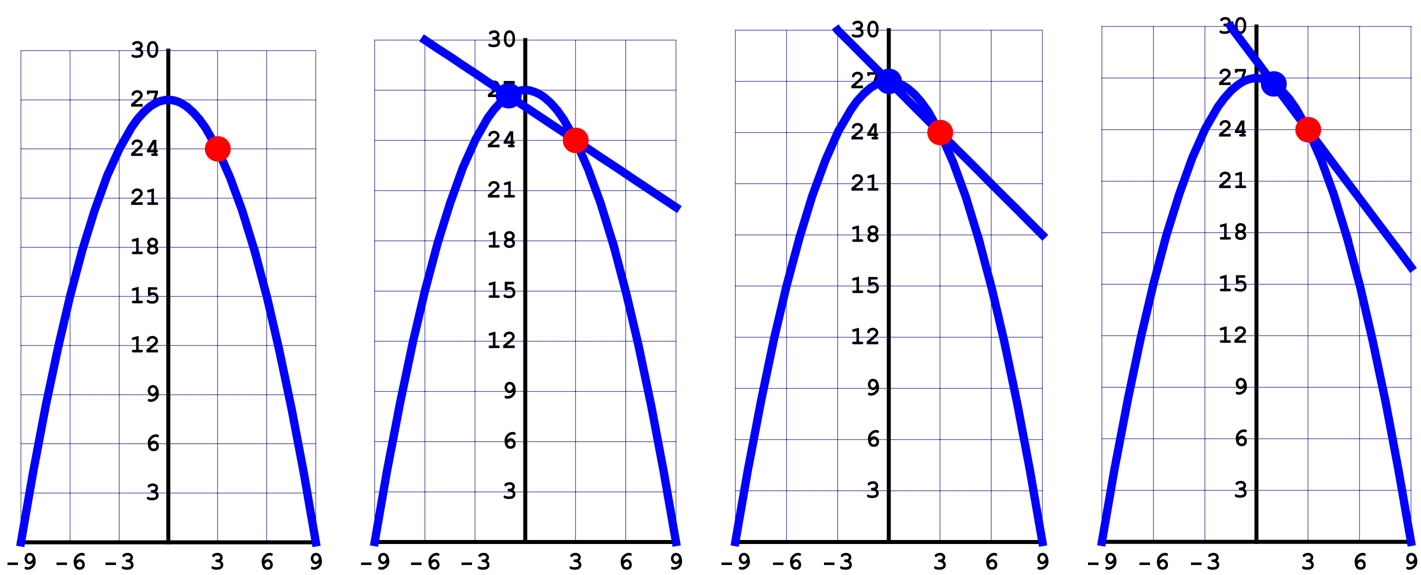
Differentiability on [a, b]

The function y = f(x) is differentiable on a closed interval [a, b] if it has a derivative at every interior point of the interval,

and if the limits $\lim_{h \to 0^+} \frac{f(a+h) - f(a)}{h}$ (the right – hand derivative at a), and

$\lim_{h \to 0^-} \frac{f(b+h) - f(b)}{h}$ (the left – hand derivative at b) exist at the endpoints.

The Paolini Building, $f(x) = 27 - \frac{x^2}{3}$

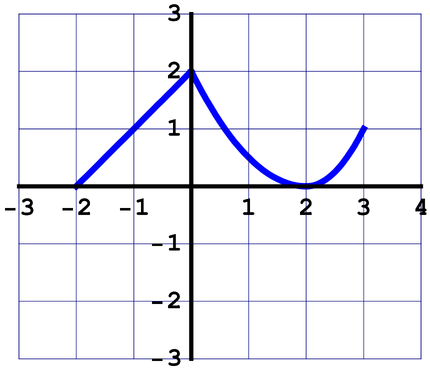


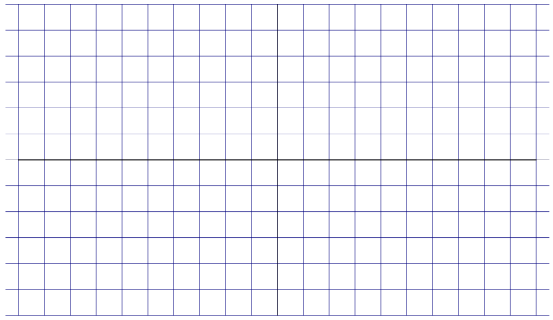
1. Use the primary definition of the derivative to find the derivative of $f(x) = x^2 + x$ at $x = -2$

2. Use the secondary definition of the derivative to find the derivative of $f(x) = x^2 + x$ at $x = -2$

For problems 3 – 6, given the graph of $f(x)$, sketch the graph of $f'(x)$.

3.





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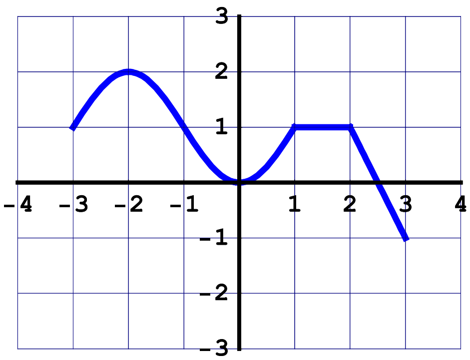

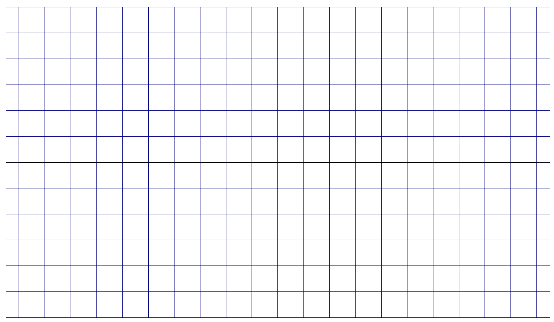

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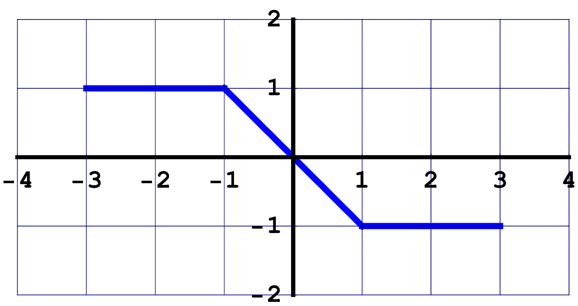

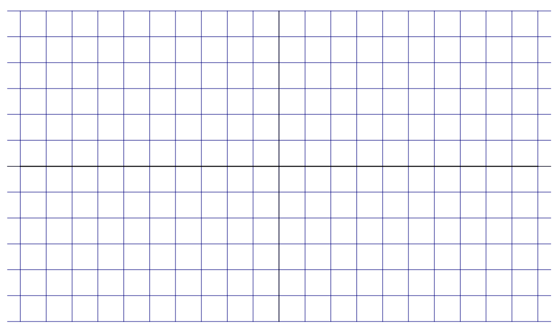

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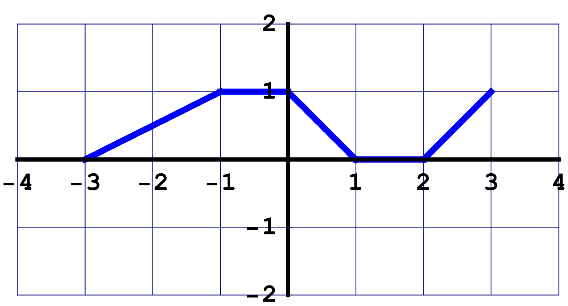

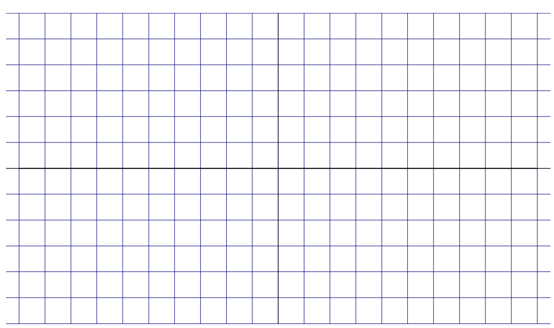

7. For the function $f(x) = \begin{cases} 2x, & x \le -1 \\ -x^2 + x, & x > -1 \end{cases}$ find the left – hand and right – hand derivatives at $x = -1$.