3.3 Rules for Differentiation

Formulas

(1) $\frac{d}{dx}(c) = 0$

(2) $\frac{d}{dx}(x^n) = n\,x^{n-1}$

(3) $\frac{d}{dx}(c\,u) = c\frac{d}{dx}(u) = c\frac{du}{dx}$

(4) $\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$

(5) $\frac{d}{dx}(u\,v) = u\frac{dv}{dx} + v\frac{du}{dx}$

(6) $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

(7) $\frac{d}{dx}\left(\frac{1}{u}\right) = \frac{-\frac{du}{dx}}{u^2}$

First, Second, and Higher Derivative Notations

$f'(x),\quad f''(x),\quad f^{(5)}(x)$ $\qquad y',\ y'',\ y^{(5)}$ $\qquad \frac{dy}{dx},\ \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2},\ \frac{d^5y}{dx^2}$

For problems 1 – 4, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

1. $f(x) = x^4 - x^2$

$$\frac{dy}{dx} = 4x^3 - 2x$$
$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{d}{dx}y\right) = 12x^2 - 2$$

2. $f(x) = 3x^3 + 5x - 7$

$$\frac{dy}{dx} = 9x^2 + 5$$
$$\frac{d^2y}{dx} = 18x$$

3. $f(x) = 4x^{-3} - 6x^{-2} + x^{-1}$

$$\frac{dy}{dx} = -12x^{-4} + 12x^{-3} - x^{-2}$$
$$\frac{d^2y}{dx} = 48x^{-5} - 36x^{-4} + 2x^{-3}$$

4. $f(x) = 2x\sqrt{x} - 9\sqrt{x} + 2$

$f(x) = 2x\sqrt{x} - 9\sqrt{x} + 2 = 2x^{\frac{3}{2}} - 9x^{\frac{1}{2}} + 2$

$$\frac{dy}{dx} = 3x^{\frac{1}{2}} - \frac{9}{2}x^{-\frac{1}{2}}$$
$$\frac{d^2y}{dx^2} = \frac{3}{2}x^{-\frac{1}{2}} + \frac{9}{4}x^{-\frac{3}{2}}$$

For problems 5 – 7, find $f'(x)$.

5. $f(x) = (x^3 - 1)\left(\frac{1}{x^2} + 2x\right)$

$f(x) = (x^3 - 1)(x^{-2} + 2x)$

$$f'(x) = (x^3 - 1)(-2x^{-3} + 2) + (x^{-2} + 2x)(3x^2)$$

6. $f(x) = \frac{(x+3)(x^2-1)}{x+2} = \frac{x^3 + 3x^2 - x - 3}{x+2}$

$$f'(x) = \frac{(x+2)(3x^2 + 6x - 1) - (x^3 + 3x^2 - x - 3)(1)}{(x+2)^2}$$

7. $f(x) = \frac{5x + 3\sqrt{x}}{8 - x^2}$

$$f'(x) = \frac{(8 - x^2)\left(5 + \frac{3}{2}x^{-\frac{1}{2}}\right) - \left(5x + 3\sqrt{x}\right)(-2x)}{(8 - x^2)^2}$$

8. Suppose that u and v are functions that are differentiable at $x = -3$, and that $u(-3) = -1$, $u'(-3) = 2$, $v(-3) = 3$, and $v'(-3) = 4$, find the following at $x = -3$ (a) $\frac{d}{dx}(uv)$ (b) $\frac{d}{dx}(6u - 2v)$ (c) $\frac{d}{dx}\left(\frac{v}{u}\right)$

(a) $u'v + uv' = 2(3) + (-1)(4) = \boxed{2}$

(b) $6u' - 2v' = 6(2) - 2(4) = \boxed{4}$

(c) $\dfrac{uv' - vu'}{u^2} \rightarrow \dfrac{(-1)(4) - (3)(2)}{(-1)^2} = \boxed{-10}$

9. Find the equation of the normal line to the curve $y = x - \sqrt{x} + 1$ at the point $(4, 3)$.

$$\frac{dy}{dx} = 1 - \frac{1}{2}x^{-\frac{1}{2}} = 1 - \frac{1}{2}(4)^{-\frac{1}{2}} = 1 - \frac{1}{4} = \frac{3}{4}$$

$$f'(4) = 1 - \frac{1}{2}(4)^{-\frac{1}{2}} = 1 - \frac{1}{4} = \frac{3}{4}$$

Since we are seeking the normal line, the slope is the negative reciprocal or $-\frac{4}{3}$

$$\boxed{y - 3 = -\frac{4}{3}(x - 4)}$$

10. Find an equation for the tangent line to the curve $y = \dfrac{x^2 + 3}{2x}$ at the point $(1, 2)$.

$$y' = \frac{2x(2x) - (x^2 + 3)(2)}{4x^2} \qquad f'(1) = \frac{4 - 8}{4} = -1$$

$$\boxed{y - 2 = -1(x - 1)}$$