3.3 Rules for Differentiation

Formulas

(1) $\frac{d}{dx}(c) = 0$

(2) $\frac{d}{dx}(x^n) = n x^{n-1}$

(3) $\frac{d}{dx}(c u) = c \frac{d}{dx}(u) = c \frac{du}{dx}$

(4) $\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$

(5) $\frac{d}{dx}(u v) = u \frac{dv}{dx} + v \frac{du}{dx}$

(6) $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

(7) $\frac{d}{dx}\left(\frac{1}{u}\right) = \frac{-\frac{du}{dx}}{u^2}$

First, Second, and Higher Derivative Notations

$f'(x)$, $f''(x)$, $f^{(5)}(x)$ $\qquad$ y', y'', $y^{(5)}$ $\qquad$ $\frac{dy}{dx}$, $\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}$, $\frac{d^5y}{dx^2}$

For problems 1 – 4, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

1. $f(x) = x^4 - x^2$

2. $f(x) = 3x^3 + 5x - 7$

3. $f(x) = 4x^{-3} - 6x^{-2} + x^{-1}$

4. $f(x) = 2x\sqrt{x} - 9\sqrt{x} + 2$

For problems 5 – 7, find $f'(x)$.

5. $f(x) = (x^3 - 1)\left(\frac{1}{x^2} + 2x\right)$

6. $f(x) = \frac{(x+3)(x^2-1)}{x+2}$

7. $f(x) = \dfrac{5x + 3\sqrt{x}}{8 - x^2}$

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8. Suppose that u and v are functions that are differentiable at $x = -3$, and that $u(-3) = -1$, $u'(-3) = 2$, $v(-3) = 3$, and $v'(-3) = 4$, find the following at $x = -3$ (a) $\dfrac{d}{dx}(uv)$ (b) $\dfrac{d}{dx}(6u - 2v)$ (c) $\dfrac{d}{dx}\left(\dfrac{v}{u}\right)$

9. Find the equation of the normal line to the curve $y = x - \sqrt{x} + 1$ at the point $(4, 3)$.

10. Find an equation for the tangent line to the curve $y = \dfrac{x^2 + 3}{2x}$ at the point $(1, 2)$.