3.4 Velocity and Other Rates of Change

Instantaneous Rate of Change

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Velocity, Speed, and Acceleration

If the position function is described by $s(t)$, then

velocity $= v(t) = s'(t)$ speed $= |v(t)| = |s'(t)|$ acceleration $= a(t) = v'(t) = s''(t)$

Free – fall Constants on Earth

English units $\rightarrow$ $s = -16t^2$ $a = -32 \dfrac{\text{ft}}{\text{sec}^2}$ Metric units $\rightarrow$ $s = -4.9t^2$ $a = -9.8 \dfrac{\text{m}}{\text{sec}^2}$

Marginal Cost

Marginal Cost is the derivative of the Cost function.

---

1. A particle moves along a line so that its position at any time $t \geq 0$ is given by $s(t) = t^2 - t - 4$, where s is measured in meters and t is measured in seconds. Find (a) the displacement in the first 3 seconds (b) the average velocity in the first 3 seconds (c) the acceleration at $t = 3$ seconds (d) where the particle is when s is a minimum (e) the velocity when $t = 3$ seconds

(a) $s(3) - s(0) = 2 - (-4) = \boxed{6 \text{ meters}}$ (b) average velocity $= \dfrac{6 \text{ meters}}{3 \text{ seconds}} = \boxed{2 \dfrac{\text{m}}{\text{sec}}}$

(c) $v(t) = 2t - 1$ $a(t) = 2$ $a(3) = \boxed{2 \dfrac{\text{m}}{\text{sec}^2}}$

(d) find where the derivative is 0 $2t - 1 = 0$ $t = \dfrac{1}{2}$ second, and check the endpoints so

$s(0) = -4$ meters $s(1) = -4$ meters $s\left(\dfrac{1}{2}\right) = -4\dfrac{1}{4}$ meters and the particle is at a minimum when

$\boxed{t = \dfrac{1}{2} \text{ seconds, with } s\left(\dfrac{1}{2}\right) = -4\dfrac{1}{4} \text{ meters}}$ (e) $s(t) = 2t - 1$ so $v(3) = \boxed{5 \dfrac{\text{m}}{\text{sec}}}$

---

2. A calculus book thrown vertically upward on the planet Zorgon at a velocity of $27 \dfrac{\text{m}}{\text{sec}}$ reaches a height of $s = 27t - 0.6t^2$ meters in t seconds. (a) Find the book's velocity and acceleration as functions of time (b) Find how long it takes the book to reach its highest point (c) Find how high the book goes (d) Find how long the book is aloft

(a) $v(t) = 27 - 1.2t \dfrac{\text{m}}{\text{sec}}$ $a(t) = \boxed{-1.2 \dfrac{\text{m}}{\text{sec}^2}}$

(b) Find when the velocity is 0 $\rightarrow$ $27 - 1.2t = 0$ $27 = \dfrac{6}{5}t$ $t = \dfrac{135}{6} = \boxed{22.5 \text{ seconds}}$

(c) $s(22.5) = 27(22.5) - 0.6(22.5)^2 = \boxed{303.75 \text{ meters}}$ (d) using symmetry $\rightarrow$ (22.5 seconds)(2) $= \boxed{45 \text{ seconds}}$

---

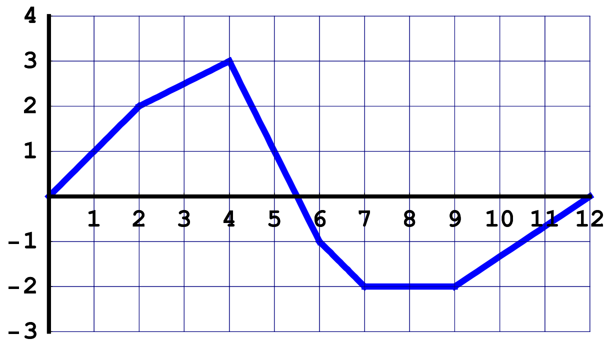
3. Suppose it costs $c(x) = x^3 - 3x^2 + 8x + 4$ dollars to produce x widgets when 5 to 15 widgets are produced. Find the marginal cost for 7 widgets.

$c'(x) = 3x^2 - 6x + 8$ when $5 \leq x \leq 15$ substitute in 7 widgets, or $x = 7$ so

$c'(7) = 3(7)^2 - 6(7) + 8 = \boxed{113 \text{ dollars}}$

---

4. The following graph relates time and velocity for a particle moving along a coordinate line

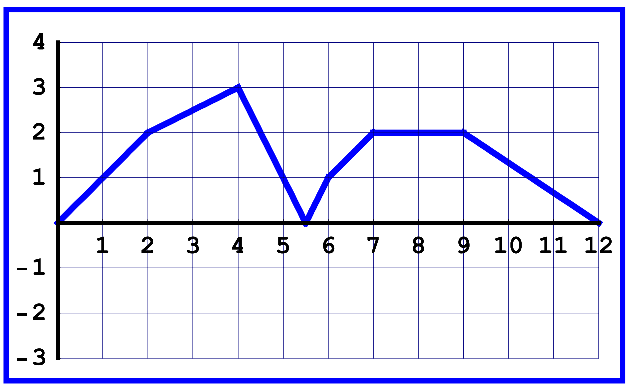


(a) find when the particle reverses direction (b) find when the body is moving at a constant speed
(c) graph the particle′s speed (d) graph the particle′s acceleration

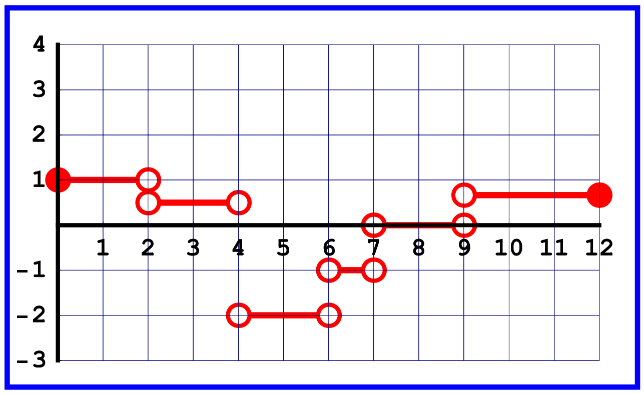
(a) look for where the velocity changes from positive to negative, or from negative to positive → at **t = 5.5**

(b) from 7 to 9, or **on the interval [7, 9]**

(c)



(d)



5.

t (seconds)	0	3	6	9	12	15	18	21
s (feet)	0	8	12	15	16	14	21	13

Given the table above, relating time and position, approximate the velocity and acceleration at time $t = 9$ seconds.

Approximate velocity → use a symmetric difference quotient, with the data points on either side of 9 seconds, so that

$$v(9) \approx \frac{(16 - 12)\text{ feet}}{(12 - 6)\text{ seconds}} \quad \text{so} \quad \boxed{v(9) \approx \frac{2}{3}\,\frac{\text{ft}}{\text{sec}}}$$

Approximate acceleration → approximate the velocity on either side, then use a symmetric difference quotient, so

$$v(7.5) \approx \frac{(15 - 12)\text{ feet}}{(9 - 6)\text{ seconds}} = 1\,\frac{\text{ft}}{\text{sec}} \quad \text{and} \quad v(10.5) \approx \frac{(16 - 15)\text{ feet}}{(12 - 9)\text{ seconds}} = \frac{1}{3}\,\frac{\text{ft}}{\text{sec}} \quad \text{so}$$

$$a(9) \approx \frac{\left(\frac{1}{3} - 1\right)\frac{\text{ft}}{\text{sec}}}{(10.5 - 7.5)\text{ sec}} = \boxed{\frac{-2}{9}\,\frac{\text{ft}}{\text{sec}^2}}$$