3.5 Derivatives of Trigonometric Functions

Trigonometric Derivatives

$D_x \sin x = \cos x$ $\qquad$ $D_x \cos x = -\sin x$

$D_x \tan x = \sec^2 x$ $\qquad$ $D_x \cot x = -\csc^2 x$

$D_x \sec x = \sec x \tan x$ $\qquad$ $D_x \csc x = -\csc x \cot x$

Jerk

If a body's position at time t is $s(t)$, the body's jerk at time t is $j(t) = \dfrac{da}{dt} = \dfrac{d^3 s}{dt^3}$

For problems 1 – 4, find the derivative.

1. $f(x) = 5x \csc x$

$f'(x) = 5\csc x + (5x)(-\csc x \cot x)$

2. $f(x) = \dfrac{1 - \cos x}{1 + \cos x}$

$f'(x) = \dfrac{(1 + \cos x)(\sin x) - (1 - \cos x)(-\sin x)}{(1 + \cos x)^2}$

3. $f(x) = \sec x \tan x$

$f'(x) = (\sec x \tan x)\tan x + (\sec x)(\sec^2 x)$

4. $f(x) = 3x^2 \cot x - x^3 \sin x$

$f'(x) = (6x)\cot x + (3x^2)(-\csc^2 x) - (3x^2)\sin x - x^3 \cos x$

5. Find the equation of the normal line to the graph of $y = \tan x$ at the point $\left(\dfrac{\pi}{4}, 1\right)$.

$y' = \sec^2 x \quad \rightarrow \quad y'\left(\dfrac{\pi}{4}\right) = 2$ so slope is $\dfrac{-1}{2} \quad \rightarrow \quad y - 1 = \dfrac{-1}{2}\left(x - \dfrac{\pi}{4}\right)$

6. Find the equation of the tangent line to the graph of $y = x + \sin x$ at the point $\left(\dfrac{\pi}{2}, \dfrac{\pi}{2} + 1\right)$.

$y' = 1 + \cos x \quad \rightarrow \quad y'\left(\dfrac{\pi}{2}\right) = 1 \quad \rightarrow \quad y - \left(\dfrac{\pi}{2} + 1\right) = 1\left(x - \dfrac{\pi}{2}\right)$

7. For the function $f(x) = 2\sec x - \tan x$, find where the tangent line is horizontal on the interval $\left(\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right)$.

$f'(x) = 2\sec x \tan x - \sec^2 x = \sec x(2\tan x - \sec x) \qquad \sec x = 0$ or $2\tan x = \sec x$

$\dfrac{2\sin x}{\cos x} = \dfrac{1}{\cos x} \quad \rightarrow \quad \sin x = \dfrac{1}{2}$ at $x = \dfrac{\pi}{6}$,

so at the point $\left(\dfrac{\pi}{6}, \dfrac{5\sqrt{3}}{3}\right)$, the tangent line is horizontal

8. A body is moving in simple harmonic motion with position $s(t) = 3 + 3\cos t$, where t is measured in seconds and s is measured in meters. Find the body's velocity, speed, acceleration, and jerk at time $t = \dfrac{2\pi}{3}$ seconds.

$v(t) = -3\sin t$, $\quad$ speed $= |-3\sin t|$, $\quad$ $a(t) = -3\cos t$, $\quad$ $j(t) = 3\sin t$

$v\left(\dfrac{2\pi}{3}\right) = \dfrac{-3\sqrt{3}}{2}\ \dfrac{m}{sec}$ $\quad$ speed $= \dfrac{3\sqrt{3}}{2}\ \dfrac{m}{sec}$ $\quad$ $a\left(\dfrac{2\pi}{3}\right) = \dfrac{3}{2}\ \dfrac{m}{sec^2}$ $\quad$ $j\left(\dfrac{2\pi}{3}\right) = \dfrac{3\sqrt{3}}{2}\ \dfrac{m}{sec^3}$

9. Find y'' if $y = \cot x$

$y' = -\csc^2 x$ $\quad$ so $\quad$ $y'' = -2\csc x(-\csc x \cot x)$