3.6 Chain Rule

Chain Rule

$D_x[f(g(x))] = f'(g(x))\,g'(x)$ or if $y = f(u)$, $u = g(x)$ then $\dfrac{dy}{dx} = \dfrac{dy}{du}\,\dfrac{du}{dx}$

Derivative of a Parametrically Defined Plane Curve

For a parametrically defined plane curve, if $x = f(t)$ and $y = g(t)$, then $\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = \dfrac{g'(t)}{f'(t)}$

For problems 1 – 8, find $\dfrac{dy}{dx}$.

1. $y = \tan u, \quad u = 3x^2$

2. $y = \sqrt[3]{u}, \quad u = x^2 + 5x$

3. $y = -2x\cot\left(\dfrac{\sqrt{x}}{3}\right)$

4. $y = (3x^2 - x + 2)(x^3 + 5x)^4$

5. $y = \csc(\cot^2 x)$

6. $y = \left(\dfrac{3x + 4}{6x - 1}\right)^3$

7. $y = \sin^3(2x)\sqrt{\cos(3x)}$

8. $y = \cos^2\sqrt{3 - 7x}$

For problems 9 – 12, find the equation of the tangent line at the given value of t.

9. $x = \sec t, \quad y = \tan t, \quad t = \dfrac{\pi}{4}$

10. $x = 3t^2 - 6t, \quad y = \sqrt{t}, \quad t = 4$

11. $x = \cos^3 t, \quad y = \sin^2 t, \quad t = \dfrac{-\pi}{6}$

12. $x = t^3 + 1, \quad y = t^2 - 2t, \quad t = -2$

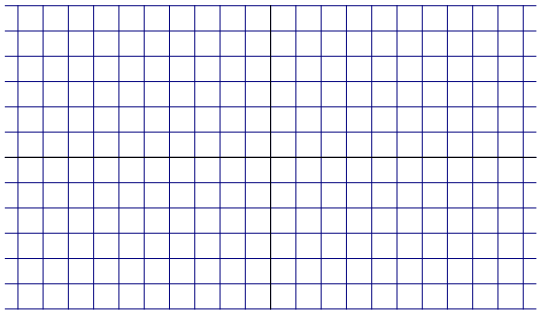
13. Let $s = \csc t$. Find $\dfrac{ds}{dy}$ when $t = \dfrac{2\pi}{3}$ and $\dfrac{dt}{dy} = \dfrac{1}{2}$.

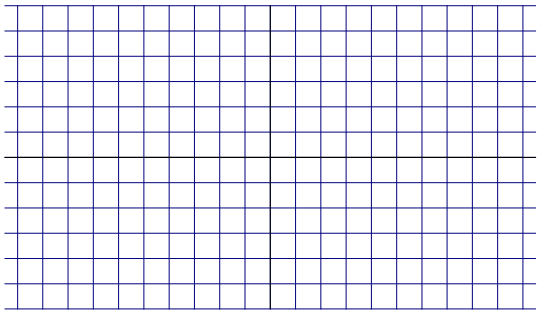
For problems 14 – 19, compare $\lim\limits_{x \to a^+} f'(x)$ and $\lim\limits_{x \to a^-} f'(x)$ and $\lim\limits_{h \to 0^+} \dfrac{f(a+h) - f(a)}{h}$ and $\lim\limits_{h \to 0^-} \dfrac{f(a+h) - f(a)}{h}$

14. $f(x) = 1, \quad x \neq 1$

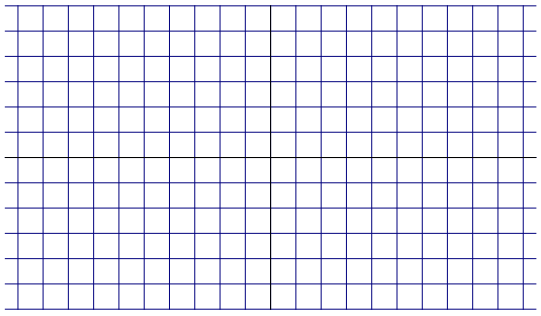
$2, \quad x = 1 \qquad a = 1$

15. $f(x) = \dfrac{1}{x - 1} \qquad a = 1$

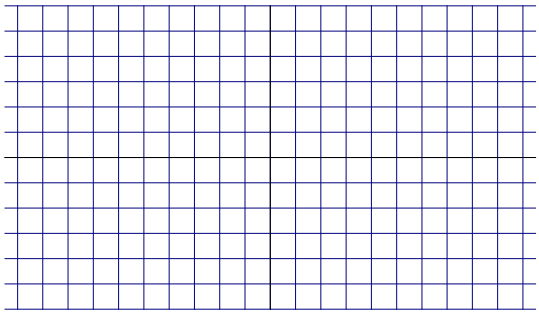




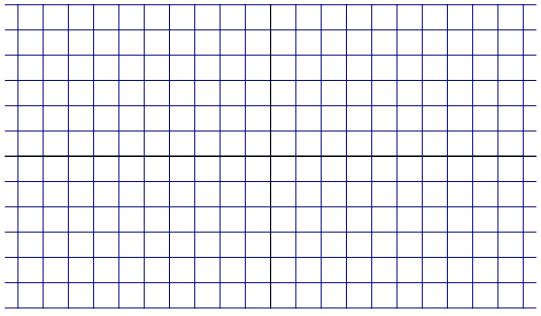
16. $f(x) = x^{\frac{2}{3}}$ $a = 0$



17. $f(x) = |x - 1|$ $a = 1$



18. $f(x) = \dfrac{|x - 1|}{x - 1}$ $a = 1$



19. $f(x) = 2x, \quad x < 1$

$x^2, \quad x \geq 1, \quad a = 1$