

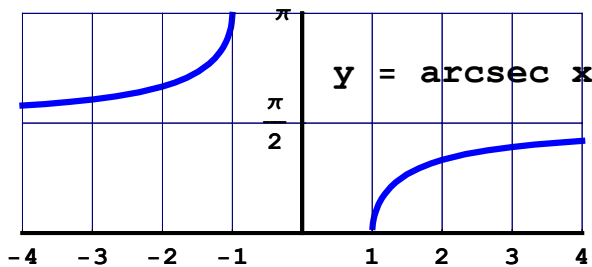
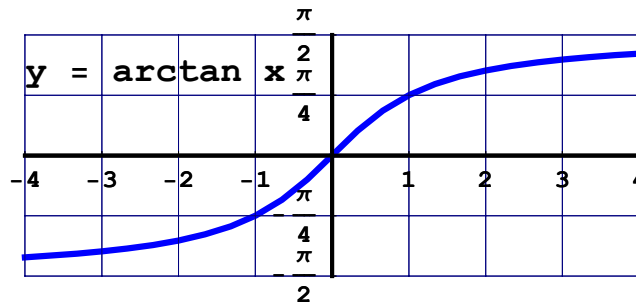
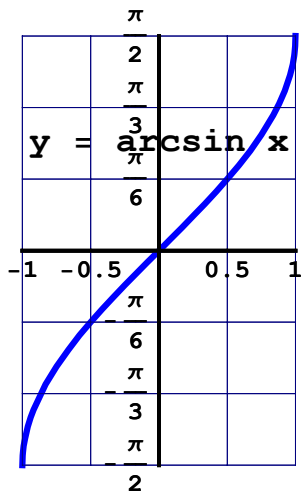
3.8 Derivatives of Inverse Trigonometric Functions

Inverse Trigonometric Function Derivatives

$$\frac{d}{dx} \sin^{-1} u = \frac{1}{\sqrt{1-u^2}} \frac{du}{dx}$$

$$\frac{d}{dx} \tan^{-1} u = \frac{1}{1+u^2} \frac{du}{dx}$$

$$\frac{d}{dx} \sec^{-1} u = \frac{1}{|u| \sqrt{u^2-1}} \frac{du}{dx}$$



Inverse Cofunction Identities

$$\cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x$$

$$\cot^{-1} x = \frac{\pi}{2} - \tan^{-1} x$$

$$\csc^{-1} x = \frac{\pi}{2} - \sec^{-1} x$$

For problems 1–6, find $\frac{dy}{dx}$

1. $y = \cos^{-1}(2x^{-3})$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1-(2x^{-3})^2}} (-6x^{-4})$$

2. $y = x \cot^{-1} \sqrt{2x+3}$

$$\frac{dy}{dx} = (1) \cot^{-1} \sqrt{2x+3} + x \left(\frac{-1}{1+(\sqrt{2x+3})^2} \right) \frac{1}{2} (2x+3)^{-\frac{1}{2}} (2)$$

3. $y = \frac{1}{\sin^{-1}(3x)}$

$$\frac{dy}{dx} = \frac{\sin^{-1}(3x)(0) - (1) \left(\frac{1}{\sqrt{1-(3x)^2}} \right) (3)}{(\sin^{-1}(3x))^2}$$

4. $y = \cot^{-1} \left(\frac{1}{x} \right) - \tan^{-1} x$

$$\frac{dy}{dx} = \frac{-1}{1+\left(\frac{1}{x}\right)^2} (-x^{-2}) - \frac{1}{1+x^2} = \frac{1}{x^2+1} - \frac{1}{1+x^2} = \boxed{0}$$

$$5. y = \frac{\sec^{-1}(x+1)}{x}$$

$$\frac{dy}{dx} = \frac{x \frac{1}{|x+1|\sqrt{(x+1)^2-1}} (1) - (1)\sec^{-1}(x+1)}{x^2}$$

$$6. y = 5 \tan^{-1}(x^3 + 3x)$$

$$\frac{dy}{dx} = 5 \left(\frac{1}{1 + (x^3 + 3x)^2} \right) (3x^2 + 3)$$

7. Let $f(x) = x^5 + 2x - 1$. Find $f(1)$, $f'(1)$, $f^{-1}(2)$, and $(f^{-1})'(2)$

$$f(1) = 1 + 2 - 1 = 2$$

$$f'(x) = 5x^4 + 2$$

$$f'(1) = 7$$

$$f^{-1}(2) = 1$$

$$(f^{-1})'(2) = \frac{1}{7}$$

8. Derive the formula for the derivative of $y = \cot^{-1} x$

$$\cot^{-1} x = \frac{\pi}{2} - \tan^{-1} x \quad \rightarrow \quad \frac{d}{dx} (\cot^{-1} x) = \frac{d}{dx} \left(\frac{\pi}{2} - \tan^{-1} x \right) = \frac{-1}{1+x^2}$$

9. Find the right end behavior model for $y = \sec^{-1} x$ and $y = \sin^{-1} x$

$$y = \sec^{-1} x \quad \rightarrow \quad y = \frac{\pi}{2}$$

$$y = \sin^{-1} x \quad \rightarrow \quad \text{no right end behavior model}$$