4.1 Extreme Values of Functions

Absolute (or Global) Extreme Values

Let f be a function with domain D. Then $f(c)$ is the

(a) absolute maximum on D if and only if $f(x) \le f(c)$ for all x in D

(b) absolute minimum on D if and only if $f(x) \ge f(c)$ for all x in D

Extreme Value Theorem

If f is continuous on $[a, b]$, then f has a maximum value and a minimum value on the interval.

Local (or Relative) Extreme Values

Let c be an interior point of the domain of the function of f. Then $f(c)$ is a

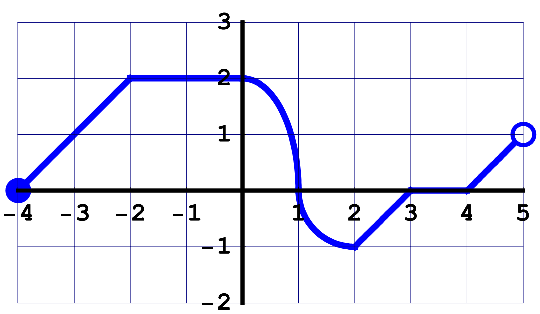
(a) local maximum at c if and only if $f(x) \le f(c)$ for all x in some open interval containing c

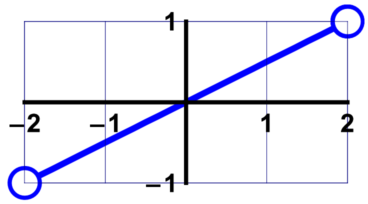
(b) local minimum at c if and only if $f(x) \ge f(c)$ for all x in some open interval containing c

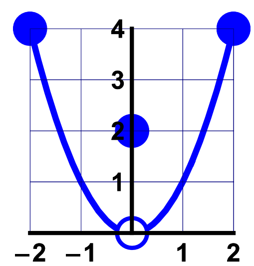
Critical Point

A point $(c, f(c))$ in the interior of the domain of a function f at which $f'(c) = 0$ or $f'(c)$ does not exist is a critical point of f.

1. for the following functions, find all local and absolute extreme values.







For problems $2 - 9$, find the extreme values of the function, and where they occur.

2. $f(x) = \dfrac{1}{x}$ on $(-4, 1)$

3. $f(x) = \sqrt[3]{x^2}$ on $[-4, 3]$

4. $f(x) = 2^x$ on $[-1, 5]$

5. $f(x) = \csc x$ on $(0, 2\pi)$

6. $y = x^3 + x^2 - 8x + 2$

7. $y = \dfrac{1}{\sqrt{4 - x^2}}$

8. $y = \dfrac{x}{x^2 + 1}$

9. $y = x^{\frac{2}{3}}(x^2 - 1)$