4.2 Mean Value Theorem

Mean Value Theorem

If $y = f(x)$ is continuous on $[a, b]$ and differentiable on (a, b), then there is at least one c on (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Increasing and Decreasing Functions

Let f be continuous on $[a, b]$ and differentiable on (a, b)

(1) If $f' > 0$ on (a, b) then f increases on $[a, b]$

(2) If $f' < 0$ on (a, b) then f decreases on $[a, b]$

Functions with the Same Derivative Differ by a Constant

If $f'(x) = g'(x)$ at each point of an interval I, then there is a constant C such that $f(x) = g(x) + C$ for all x in I.

Antiderivative

A function $F(x)$ is an antiderivative of a function $f(x)$ if $F'(x) = f(x)$ for all x in the domain of f. The process of finding an antiderivative is antidifferentiation.

For problems 1 – 4, find (a) the local extrema, (b) the intervals on which the function is increasing, and (c) the intervals on which the function is decreasing.

1. $f(x) = x^2 + 2x - 15$

2. $f(x) = \left(\frac{1}{2}\right)^x$

3. $f(x) = \frac{5}{x^4}$

4. $f(x) = \frac{x}{x^2 - 9}$

For problems 5 – 8, (a) determine whether the function f satisfies the hopotheses of the Mean Value Theorem on the given interval, and (b) find all values of c that satisfy the equation $f'(c) = \dfrac{f(b) - f(a)}{b - a}$

5. $f(x) = \sqrt[3]{x + 2}$ $[-3, 6]$

6. $f(x) = \tan^{-1} x$ $[-1, \sqrt{3}]$

7. $f(x) = x + \dfrac{4}{x}$ $[-4, 1]$

8. $f(x) = \cos \dfrac{x}{2}$ $[\dfrac{-\pi}{2}, \pi]$

For problems 9 – 11, find all possible functions f with the given derivative.

9. $f'(x) = x^3 + x^2 - x$

10. $f'(x) = \dfrac{2}{\sqrt{1 - 4x^2}}$

11. $f'(x) = (\ln 5)\, 5^x$