4.3 Connecting f' and f'' with the Graph of f

First Derivative Test

Let $f(x)$ be continuous on a given open interval that contains c, a critical value

(1) If $f'(x)$ changes from $+$ to $-$ at c, then $(c, f(c))$ is a local maximum

(2) If $f'(x)$ changes from $-$ to $+$ at c, then $(c, f(c))$ is a local minimum

Concavity Test

Let $f(x)$ be continuous on a given interval (a, b)

(1) $f(x)$ is Concave Up if $f''(x) > 0$ on (a, b)

(2) $f(x)$ is Concave Down if $f''(x) < 0$ on (a, b)

Point of Inflection

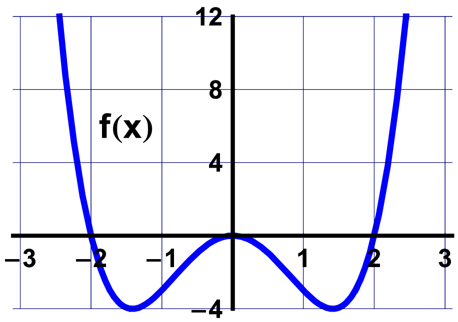
$(c, f(c))$ is a Point of Inflection if $f'(c)$ exists and $f''(x)$ changes sign at c.

Second Derivative Test

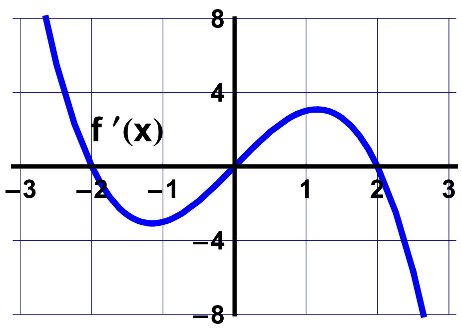
(1) If $f'(c) = 0$ and $f''(c) < 0$, then $(c, f(c))$ is a local maximum

(2) If $f'(c) = 0$ and $f''(c) > 0$, then $(c, f(c))$ is a local minimum

1. Use the graph of the function f to estimate where (a) f' and (b) f'' are zero, positive, and negative.



2. Use the graph of f' to estimate where the function f is (a) increasing or (b) decreasing, and (c) estimate where f has local extreme values.



For the following problems, use analytic methods to find the intervals on which the function is (a) increasing, (b) decreasing, (c) concave up, (d) concave down, then find any (e) local extreme values, and (f) inflection points

9. $y = 2x^4 - 4x^2 + 1$

23. $y = \tan^{-1} x$

25. $y = x^{\frac{1}{3}}(x - 4)$

28. $y = \dfrac{x}{x^2 + 1}$