4.5 Linearization and Newton′s Method

Linearization

If f is differentiable at $x = a$, then $L(x) = f(a) + f'(a)(x - a)$ is the linearization of f at a.

Newton′s Method

1. Guess an approximation to the solution of $f(x) = 0$
2. Find successive approximations with $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

Differentials

Assume $y = f(x)$ is differentiable. dx is an independent variable, and $dy = f'(x)\,dx$

□	Exact	Estimated
Absolute Change	$\Delta y = \Delta f = f(a + dx) - f(a)$	$dy = df = f'(a)\,dx$
Relative Change	$\frac{\Delta y}{f(a)} = \frac{\Delta f}{f(a)}$	$\frac{dy}{f(a)} = \frac{df}{f(a)}$
Percentage Change	$100\,\frac{\Delta y}{f(a)} = 100\,\frac{\Delta f}{f(a)}$	$100\,\frac{dy}{f(a)} = 100\,\frac{df}{f(a)}$

For problems 1 and 2, find the linearization $L(x)$ of $f(x)$ at $x = a$

1. $f(x) = 2\sin\theta + \cos\theta, \quad x = \dfrac{5\pi}{6}$

2. $f(x) = \sqrt{1 + 2x}, \quad x = 2$

3. Choose a linearization at a nearby value, so that the function can be approximated at $x = a$ if

$f(x) = \dfrac{2x}{x + 2}, \quad a = 2.1$

For problems 4 and 5, use Newton′s Method to estimate the solution to the equation on the given interval.

4. $x^3 - 36x - 84 = 0$ $[6, 8]$

5. $x^5 - 2x^2 + 4 = 0$ $[-2, -1]$

For problems 6 and 7, find (a) Δy, and (b) dy, and (c) $\Delta y - dy$.

6. $f(x) = x^2 + 3x$, $a = 1$, $\Delta x = 0.1$

7. $f(x) = x(\ln(x))$, $a = e$, $\Delta x = -0.1$

8. Write a differential formula that estimates the change in volume, $V = \frac{4}{3}\pi r^3$, when the radius changes from a to $a + dr$

9. About how accurately should you measure the side of a square to be sure of calculating the area to within 2% of its true value?