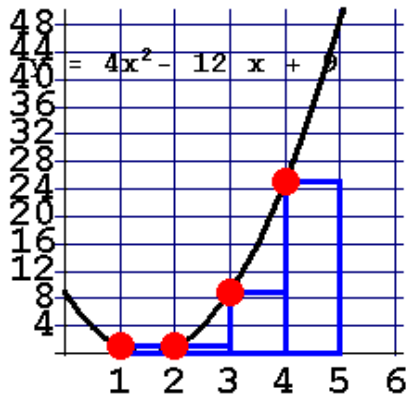
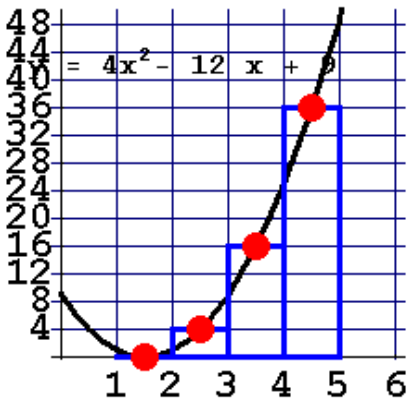
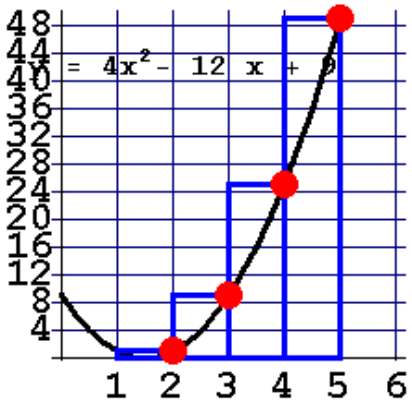
5.1 Approximating the area underneath a curve

1. **Approximate the area under the curve $f(x) = 4x^2 - 12x + 9$ on the interval $[1, 5]$. Use 4 equal subintervals, and the methods LRAM, MRAM, and RRAM.**





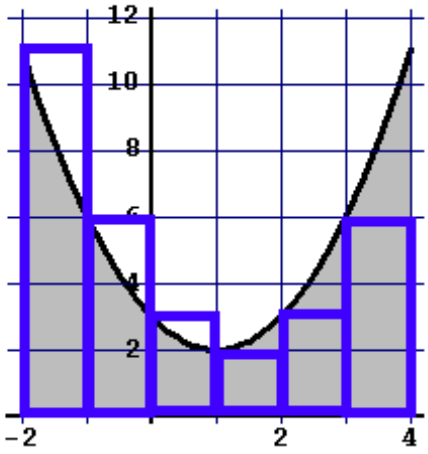


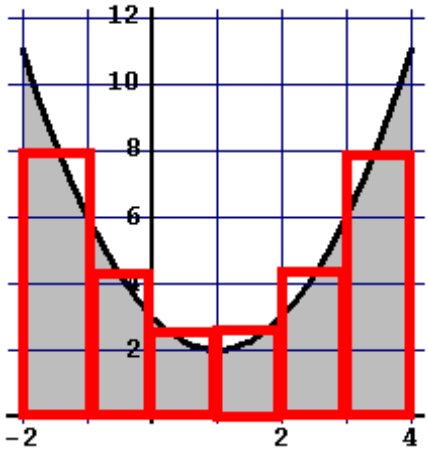
LRAM $= 1(1) + 1(1) + 1(9) + 1(25) =$ **36**

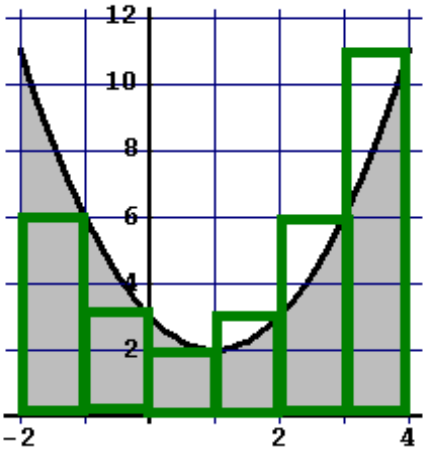
MRAM $= 1(0 + 4 + 16 + 36) =$ **56**

RRAM $= 1(1 + 9 + 25 + 49) =$ **84**

2. **Approximate the area under the curve $f(x) = x^2 - 2x + 3$ on the interval $[-2, 4]$. Use 6 equal subintervals, and the methods LRAM, MRAM, and RRAM.**





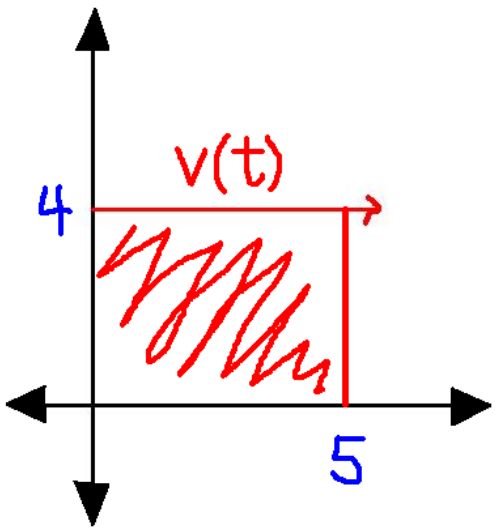


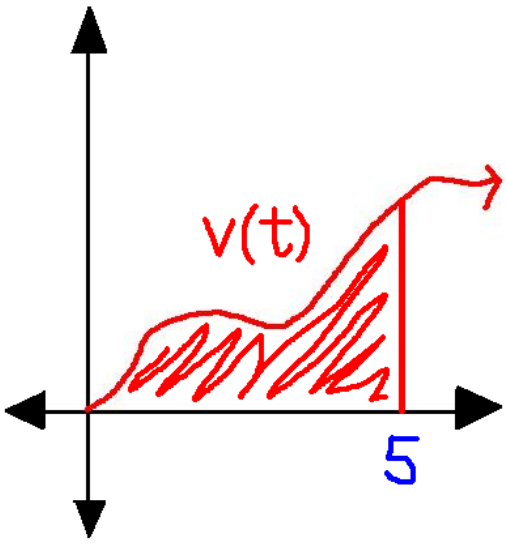
LRAM $= 1(11 + 6 + 3 + 2 + 3 + 6) =$ **31**

MRAM $= 1(f(-1.5) + f(-0.5) + f(0.5) + f(1.5) + f(2.5) + f(3.5)) =$ **$29\frac{1}{2}$**

RRAM $= 1(6 + 3 + 2 + 3 + 6 + 11) =$ **31**

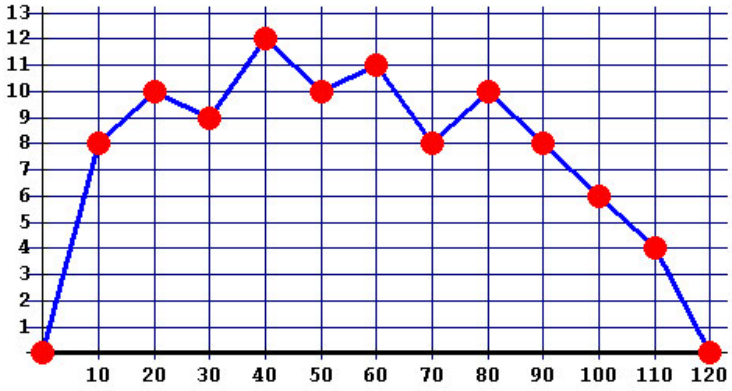
Accumulated velocity represents the displacement





3. You decide you need some exercise, so you head out to Rancho San Antonio to get a run in. You only run for 2 minutes, but that's better than nothing, right? Below is a table of your velocity at 10 – second intervals. Use LRAM and RRAM to approximate how far you ran.

Time (sec)	Velocity (ft/sec)
0	0
10	8
20	10
30	9
40	12
50	10
60	11
70	8
80	10
90	8
100	6
110	4
120	0



LRAM = $10(0 + 8 + 10 + 9 + 12 + 10 + 11 + 8 + 10 + 8 + 6 + 4)$ = **960 feet**

RRAM = $10(8 + 10 + 9 + 12 + 10 + 11 + 8 + 10 + 8 + 6 + 4 + 0)$ = **960 feet**

4. You decide to take your car out for a long drive to San Francisco, over the Golden Gate, and back. Below is a table of your velocity at 20 minute intervals. Approximate how far you travelled using LRAM and RRAM.

Time (mins)	Velocity (miles/hr)
0	0
20	60
40	56
60	62
80	58
100	60
120	64
140	55
160	60

$$\text{RRAM} = \frac{1}{3}(60 + 56 + 62 + 58 + 60 + 64 + 55 + 60) = \boxed{\frac{475}{3} \text{ miles}}$$

$$\text{LRAM} = \frac{1}{3}(0 + 60 + 56 + 62 + 58 + 60 + 64 + 55) = \boxed{\frac{415}{3} \text{ miles}}$$

5. Approximate the volume of a hemisphere with radius 5 feet by using $f(x) = \sqrt{25 - x^2}$ on the interval $[-5, 0]$. This will give us a quarter circle. Now, rotate the quarter circle around the x – axis, to obtain a hemisphere. Use 5 intervals with RRAM to approximate the volume.

$$\text{RRAM} = \pi\left(\sqrt{25 - (-4)^2}\right)^2(1) + \pi\left(\sqrt{25 - (-3)^2}\right)^2(1) + \pi\left(\sqrt{25 - (-2)^2}\right)^2(1) +$$

$$\pi\left(\sqrt{25 - (-1)^2}\right)^2(1) + \pi\left(\sqrt{25 - (0)^2}\right)^2(1) = \pi(9 + 16 + 21 + 24 + 25) = \boxed{95\pi \text{ ft}^3}$$