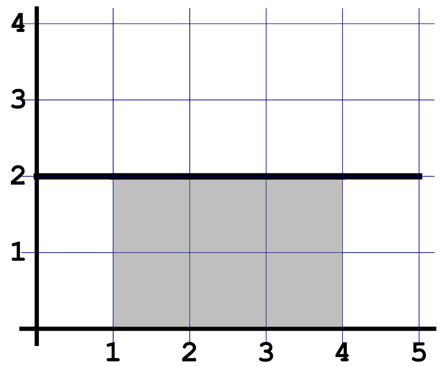
Definite Integrals

Definitions of the Definite Integral

$$\int_a^b f(x)\,dx = \lim_{\|P\|\to 0} \sum_{k=1}^{n} f(c_k)\,\Delta x_k \qquad \text{or} \qquad \int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} f\left(a + \frac{b-a}{n}k\right)\left(\frac{b-a}{n}\right)$$

Think of the formulas above as the "area under the curve" formulas.



First Shortcut $\quad \int_a^b c\,dx = c\,(b - a)$

$$\int_1^4 2\,dx = 2(4-1) = \boxed{6}$$

1. $\int_1^6 4\,dx$

$4(6-1) = \boxed{20}$

2. $\int_{-2}^{3} (-5)\,dx$

$-5(3-(-2)) = \boxed{-25}$

3. $\int_7^5 \left(\frac{-\pi}{2}\right) dx$

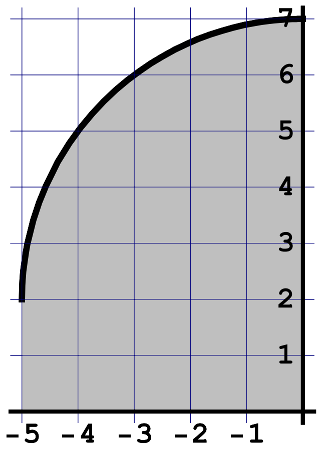
$-\frac{\pi}{2}(5-7) = \boxed{\pi}$

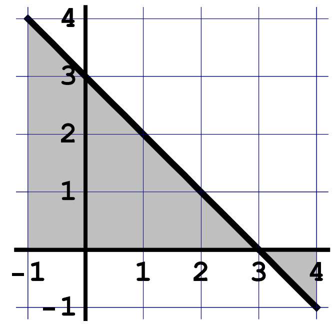
4. $\lim_{\|P\|\to 0} \sum_{k=1}^{n} (c_k^2 - 4c_k + 2)\,\Delta x_k$, where P is any partition of $[-3, 2]$

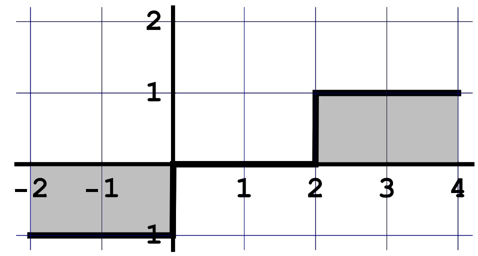
$$\boxed{\int_{-3}^{2} (x^2 - 4x + 2)\,dx}$$

5. $\lim_{\|P\|\to 0} \sum_{k=1}^{n} \log_6(5c_k - 3)\,\Delta x_k$, where P is any partition of $[2, 4]$

$$\boxed{\int_2^4 \log_6(5x - 3)\,dx}$$







6. $\displaystyle\int_{-5}^{0} \left(2 + \sqrt{25 - x^2}\right)\, dx$

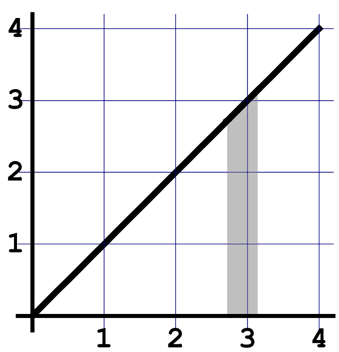
$= 4\,(2) + \dfrac{1}{4}\,\pi\,(5^2) = \boxed{8 + \dfrac{25\,\pi}{4}}$

7. $\displaystyle\int_{-1}^{4} (-x + 3)\, dx$

$= \dfrac{1}{2}\,(4)\,(4) - \dfrac{1}{2}\,(1)\,(1) = \boxed{7\,\dfrac{1}{2}}$

8. $\displaystyle\int_{-2}^{4} [\![\tfrac{1}{2}\, x]\!]\, dx$

$= -2 + 0 + 2 = \boxed{0}$



9. $\displaystyle\int_{e}^{\pi} s\, ds$

$= (\pi - e)\,\dfrac{1}{2}\,(\pi + e)$

$= \boxed{\dfrac{\pi^2 - e^2}{2}}$

10. $\displaystyle\int_{a}^{b} -3\,y\, dy$ where $0 < a < b$

$= (b - a)\,\dfrac{1}{2}\,(-3\,a - 3\,b)$

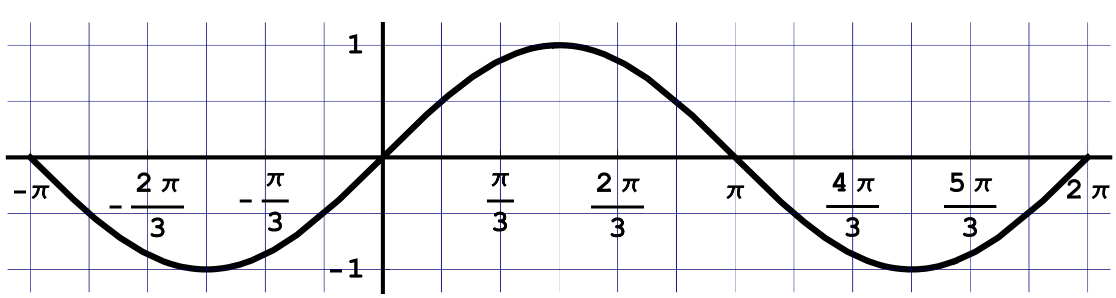
$= \boxed{\dfrac{3}{2}\,(a^2 - b^2)}$

11. $\displaystyle\int_{b}^{2\,b} \sqrt{3}\, r\, dr$ where $0 < b$

$= (2\,b - b)\,\dfrac{1}{2}\left(\sqrt{3}\,(2\,b) + \sqrt{3}\,(b)\right)$

$= \dfrac{b}{2}\left(3\,\sqrt{3}\,b\right) = \boxed{\dfrac{3\,\sqrt{3}}{2}\,b^2}$

Assuming that $\displaystyle\int_{0}^{\frac{\pi}{2}} \sin x\, dx = 1$, evaluate the following integrals



12. $\int_0^{\frac{3\pi}{2}} \sin x \, dx$

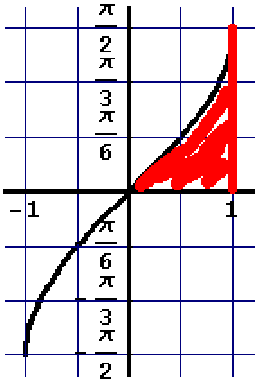
$= 1 + 1 - 1 = \boxed{1}$

13. $\int_{\frac{\pi}{6}}^{\frac{7\pi}{6}} \sin\left(x - \frac{\pi}{6}\right) dx$

$= 1 + 1 = \boxed{2}$

14. $\int_0^{\frac{\pi}{2}} (1 - \sin x) \, dx$

$= \boxed{\frac{\pi}{2} - 1}$



15. $\int_0^1 \sin^{-1} x \, dx$

$\boxed{\frac{\pi}{2} - 1}$