5.3 Definite Integrals and Antiderivatives

Rules for Definite Integrals

1. $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$

2. $\int_a^a f(x)\,dx = 0$

3. $\int_a^b k\,f(x)\,dx = k\int_a^b f(x)\,dx$ for any real k

4. $\int_a^b (f(x) \pm g(x))\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

5. $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$

6. $(\text{min of } f)(b - a) \le \int_a^b f(x)\,dx \le (\text{max of } f)(b - a)$

7. If $f(x) \ge g(x)$ on $[a, b]$, then $\int_a^b f(x)\,dx \ge \int_a^b g(x)\,dx$. If $f(x) \ge 0$ on $[a, b]$, then $\int_a^b f(x)\,dx \ge 0$

Assume that $\int_{-2}^1 h(x)\,dx = 3$, $\int_5^1 h(x)\,dx = 4$, and $\int_1^5 g(x)\,dx = -6$, and find...

1. $\int_1^5 (h(x) - g(x))\,dx$

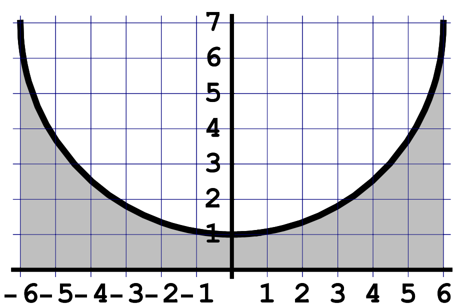
2. $\int_5^1 (-3\,g(x))\,dx$

3. $\int_1^5 (2\,g(x) - 4\,h(x))\,dx$

4. $\int_5^{-2} \left(\frac{h(x)}{3}\right) dx$

Don't forget your geometry skills...

5. $\int_{-6}^6 \left(7 - \sqrt{36 - x^2}\right) dx$



So now, let's evaluate definite integrals in a new way . . .

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x) \qquad \text{and} \qquad \int_a^x f(t)\,dt = F(x) - F(a) \quad \text{where} \quad F'(x) = f(x)$$

(we'll make use of this today, and prove it the next day in class)

That's right, to evaluate a definite integral, we can find an antiderivative!

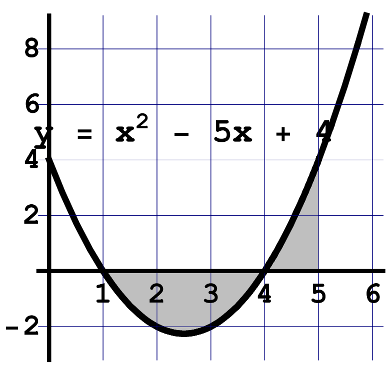
6. $\displaystyle\int_1^3 e^{2x}\,dx$

7. $\displaystyle\int_{-2}^1 \frac{2}{x+3}\,dx$

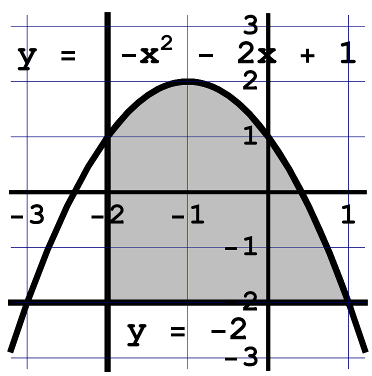
8. $\displaystyle\int_1^2 \frac{4}{x\sqrt{x^2-1}}\,dx$

9. $\displaystyle\int_{\frac{-\pi}{6}}^{\frac{\pi}{2}} (-3\sin x)\,dx$

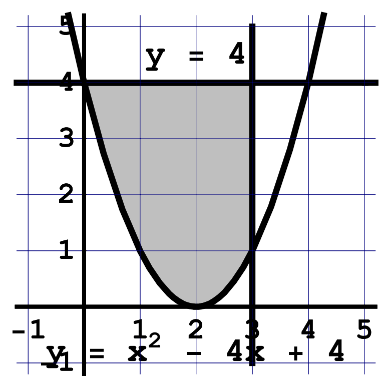
For the following problems, find the total shaded area.



10.



11.



12.

Average Value : If $f(x)$ is integrable on $[a, b]$, then $f_{av} = \dfrac{1}{b-a}\displaystyle\int_a^b f(x)\,dx$

Mean Value Theorem for Definite Integrals

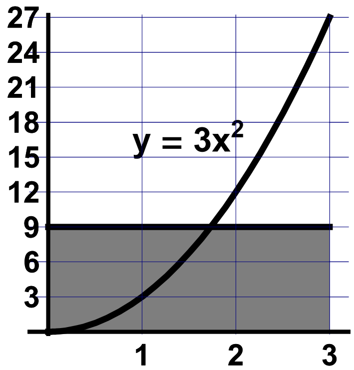
If $f(x)$ is continuous on $[a, b]$, then there exists a value c on $[a, b]$ such that $f(c) = \dfrac{1}{b-a}\displaystyle\int_a^b f(x)\,dx$

For the following problems, find the average value of the function over the given interval. Also, find the points on the interval where this average value occurs (the c values).

13. $f(x) = 3x^2$ on $[0, 3]$

$$f_{av} = \frac{1}{3-0}\int_0^3 3x^2\,dx = \frac{1}{3}\left[x^3\right]_0^3 = \frac{1}{3}(27-0) = 9$$

and so $f(c) = 3c^2 = 9$ so $c = \sqrt{3}$



14. $f(x) = \dfrac{3}{x^2}$ on $[-4, -1]$

15. $f(x) = 3\sqrt{x+1}$ on $[-1, 8]$

16. $f(x) = \begin{cases} 2x + 4 & x < 0 \\ -2x + 4 & x \geq 0 \end{cases}$ on $[-3, 3]$