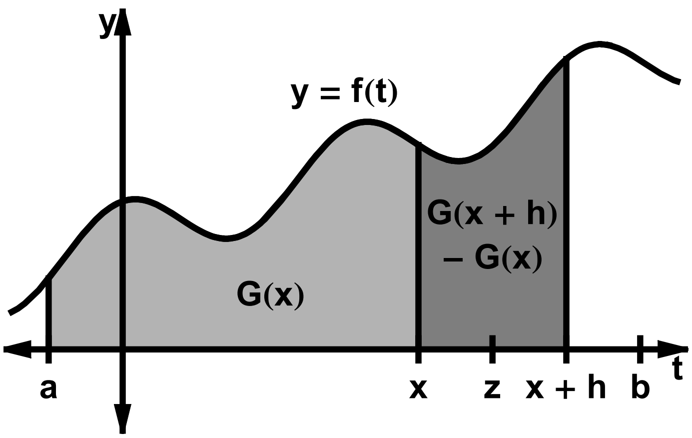
5.4 Fundamental Theorem of Calculus

I. If f is continuous on [a, b], then the function $G(x) = \int_a^x f(t)\,dt$

has a derivative at every point x in [a, b], and $\frac{dG}{dx} = \frac{d}{dx}\int_a^x f(t)\,dt = f(x)$

II. If f is continuous at every point of [a, b], and if F is any antiderivative

of f on [a, b], then $\int_a^b f(x)\,dx = F(b) - F(a)$



1. $\int_1^4 (\ln 4)\, 4^x\, dx$

2. $\int_0^{\frac{\pi}{6}} \frac{2\sin x}{\cos^2 x}\, dx$

3. $\int_{-2}^{-1} \left(x - \frac{1}{x}\right)^2 dx$

4. $\int_{-1}^{-2} \frac{2t - 7}{t^3}\, dt$

5. $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (4\sin 2\theta + 6\cos 3\theta)\, d\theta$

6. $\displaystyle\int_{-3}^{0} \frac{x^3 + 8}{x + 2}\, dx$

7. $\displaystyle\int_{0}^{\pi} \sec x\, dx$

$D_x \displaystyle\int_a^x f(t)\,dt = f(x)$ and $D_x \displaystyle\int_a^{g(x)} f(t)\,dt = f(g(x))\,g'(x)$ and $D_x \displaystyle\int_{h(x)}^{g(x)} f(t)\,dt = f(g(x))\,g'(x) - f(h(x))\,h'(x)$

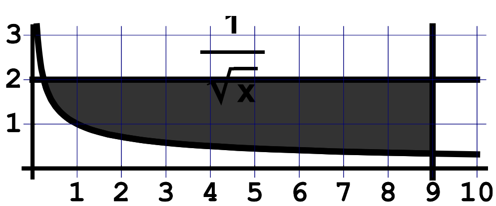
8. $D_x \displaystyle\int_0^{x^3} e^{t^2}\, dt$

9. $\dfrac{d}{dx} \displaystyle\int_{\frac{\pi}{3}}^{x} \cot s\, ds$

10. $D_x \displaystyle\int_{x^2}^{\sqrt{x}} (5t - 3)\, dt$

11. $D_t \displaystyle\int_{\sin^2 t}^{\cos(t^2)} (x^2 - 3x + 2)\, dx$

Find the area of the shaded regions

12. 

13.