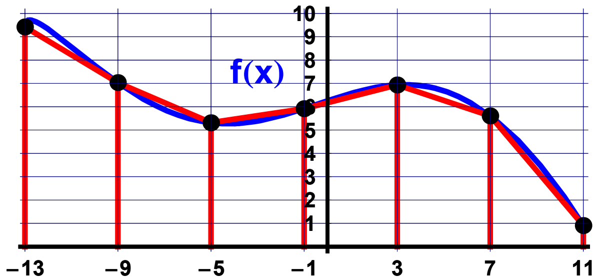
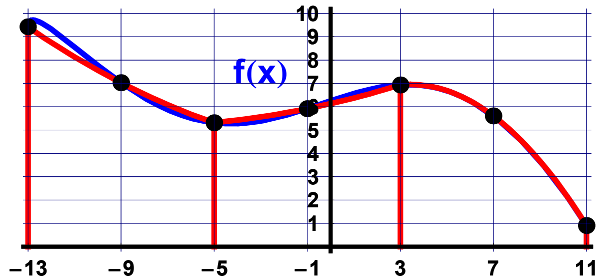
5.5 Trapezoidal Rule and Simpson′s Rule

The Trapezoidal Rule uses trapezoids to approximate the area under the curve

$$\int_a^b f(x)\,dx \approx \frac{b-a}{2n}\left(f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + \ldots + 2f(x_{n-2}) + 2f(x_{n-1}) + f(x_n) \right)$$



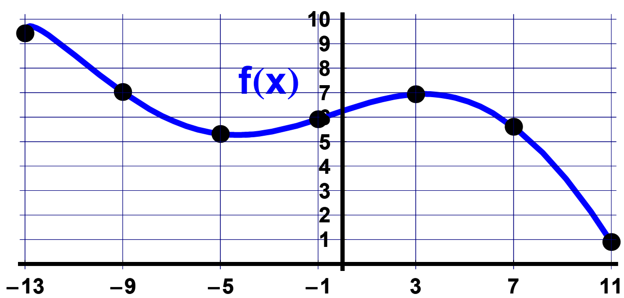


Simpson′s Rule uses quadratics to approximate the area under the curve

$$\int_a^b f(x)\,dx \approx \frac{b-a}{3n}\left(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \ldots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n) \right)$$

(n must be even for Simpson′s Rule)

1. Approximate the area for the given curve using the Trapezoidal rule and Simpson′s Rule with $n = 6$. Approximate each function value to the nearest whole number.



Trap → $\displaystyle\int_{-13}^{11} f(x)\,dx \approx \frac{11-(-13)}{2(6)}(9 + 2(7) + 2(5) + 2(6) + 2(7) + 2(6) + 1) = 2(23 + 36 + 13) = \boxed{144}$

Sim → $\displaystyle\int_{-13}^{11} f(x)\,dx \approx \frac{11-(-13)}{3(6)}(9 + 4(7) + 2(5) + 4(6) + 2(7) + 4(6) + 1) = \frac{4}{3}(37 + 48 + 25) = \frac{440}{3} = \boxed{146\,\frac{2}{3}}$

2. $\int_0^2 x^3\,dx \qquad n = 6$

$$\int_0^2 x^3\,dx \approx \frac{2-0}{2(6)}\left(f(0) + 2f\left(\frac{1}{3}\right) + 2f\left(\frac{2}{3}\right) + 2f(1) + 2f\left(\frac{4}{3}\right) + 2f\left(\frac{5}{3}\right) + f(2)\right) = \boxed{\frac{37}{9}}$$

$$\int_0^2 x^3\,dx \approx \frac{2-0}{3(6)}\left(f(0) + 4f\left(\frac{1}{3}\right) + 2f\left(\frac{2}{3}\right) + 4f(1) + 2f\left(\frac{4}{3}\right) + 4f\left(\frac{5}{3}\right) + f(2)\right) = \boxed{4}$$

3. $\int_0^{\pi} \sqrt{\sin x}\,dx \qquad n = 6$

$$\int_0^{\pi} \sqrt{\sin x}\,dx \approx \frac{\pi-0}{2(6)}\left(f(0) + 2f\left(\frac{\pi}{6}\right) + 2f\left(\frac{\pi}{3}\right) + 2f\left(\frac{\pi}{2}\right) + 2f\left(\frac{2\pi}{3}\right) + 2f\left(\frac{5\pi}{6}\right) + f(\pi)\right) \approx \boxed{2.239}$$

$$\int_0^{\pi} \sqrt{\sin x}\,dx \approx \frac{\pi-0}{2(6)}\left(f(0) + 4f\left(\frac{\pi}{6}\right) + 2f\left(\frac{\pi}{3}\right) + 4f\left(\frac{\pi}{2}\right) + 2f\left(\frac{2\pi}{3}\right) + 4f\left(\frac{5\pi}{6}\right) + f(\pi)\right) \approx \boxed{2.335}$$

4. $\int_0^6 2^{-x}\,dx \qquad n = 6$

$$\int_0^6 2^{-x}\,dx \approx \frac{6-0}{2(6)}\left(f(0) + 2f(1) + 2f(2) + 2f(3) + 2f(4) + 2f(5) + f(6)\right) \approx \boxed{1.477}$$

$$\int_0^6 2^{-x}\,dx \approx \frac{6-0}{3(6)}\left(f(0) + 4f(1) + 2f(2) + 4f(3) + 2f(4) + 4f(5) + f(6)\right) \approx \boxed{1.422}$$

5. $\int_4^{12} \ln x\,dx \qquad n = 4$

$$\int_4^{12} \ln x\,dx \approx \frac{12-4}{2(4)}\left(f(4) + 2f(6) + 2f(8) + 2f(10) + f(12)\right) \approx \boxed{16.219}$$

$$\int_4^{12} \ln x\,dx \approx \frac{12-4}{3(4)}\left(f(4) + 4f(6) + 2f(8) + 4f(10) + f(12)\right) \approx \boxed{16.272}$$