From Sums to Integrals

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} f\left(a + \frac{b-a}{n}k\right)\left(\frac{b-a}{n}\right)$$

Remember, that in this formula, $x = a + \frac{b-a}{n}k$, and $\Delta x = dx = \frac{b-a}{n}$ Look for the x first!

To be more specific, you are looking for $a + \frac{b-a}{n}k$, so something like $2 + \frac{3k}{n}$ or $\frac{2k}{n}$ or $5 + \frac{k}{n}$ or . . .

1. $\lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{k}{n}\right)^4 \left(\frac{1}{n}\right)$

2. $\lim_{n\to\infty} \left(\frac{1}{n}\right) \sum_{k=1}^{n} \frac{1}{1 + \left(\frac{k}{n}\right)^2}$

3. $\lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{2}{n}\right)\left(\left(2 + \frac{k}{n}\right)^2 + 3\left(2 + \frac{k}{n}\right)\right)$

4. $\lim_{n\to\infty} \left(\frac{6}{n}\right) \sum_{k=1}^{n} \left(\frac{n}{3k + n}\right)^2$

5. $\lim_{n\to\infty} \sum_{k=1}^{n} \left(3\left(1 + \frac{2k}{n}\right)^2 - 6\right)\left(\frac{2}{n}\right)$

6. $\lim_{n\to\infty} \left(\frac{3}{n}\right) \sum_{k=1}^{n} \frac{1}{2\left(4 + \frac{2k}{n}\right) - 7}$

7. $\lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{4}{n}\right)\left(2 + 3\left(-1 + \frac{4k}{n}\right) - \left(-1 + \frac{4k}{n}\right)^2\right)$

8. $\lim_{n\to\infty} \sum_{k=1}^{n} \cos\left(\frac{\pi}{3} + \frac{\pi k}{2n}\right)\left(\frac{2}{n}\right)$