6.1 Antiderivatives and Slope Fields

Integral Formulas

$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1$ $\qquad \int \frac{1}{x}\,dx = \ln|x| + C$ $\qquad \int e^{kx}\,dx = \frac{1}{k}e^{kx} + C$

$\int \sin kx\,dx = \frac{-1}{k}\cos kx + C$ $\qquad \int \cos kx\,dx = \frac{1}{k}\sin kx + C$

$\int \sec^2 x\,dx = \tan x + C$ $\qquad \int \csc^2 x\,dx = -\cot x + C$

$\int \sec x \tan x\,dx = \sec x + C$ $\qquad \int \csc x \cot x\,dx = -\csc x + C$

$\int f(x)\,dx = F(x) + C$ where $F'(x) = f(x)$

$\int k f(x)\,dx = k\int f(x)\,dx$ $\qquad \int (f(x) \pm g(x))\,dx = \int f(x)\,dx \pm \int g(x)\,dx$

Evaluate the integral.

1. $\int (2x^3 - x^2 + 3x - 7)\,dx$

$= \boxed{\frac{1}{2}x^4 - \frac{1}{3}x^3 + \frac{3}{2}x^2 - 7x + C}$

2. $\int \left(\sqrt{u^3} - \frac{1}{2}u^{-2} + \sqrt[3]{u}\right)du$

$= \boxed{\frac{2}{5}u^{\frac{5}{2}} + \frac{1}{2u} + \frac{3}{4}u^{\frac{4}{3}} + C}$

3. $\int \frac{8x - 5}{\sqrt[4]{x}}\,dx$

$= \int \left(8x^{\frac{3}{4}} - 5x^{\frac{-1}{4}}\right)dx$

$= \boxed{\frac{32}{7}x^{\frac{7}{4}} - \frac{20}{3}x^{\frac{3}{4}} + C}$

4. $\int \left(\cos 3x - \frac{1}{x+4} + e^{-2x}\right)dx$

$= \boxed{\frac{1}{3}\sin 3x - \ln|x+4| - \frac{1}{2}e^{-2x} + C}$

5. $\int (-3\csc 2t \cot 2t)\,dt$

$= 3\int (-\csc 2t \cot 2t)\,dt$

$= \boxed{\frac{3}{2}\csc 2t + C}$

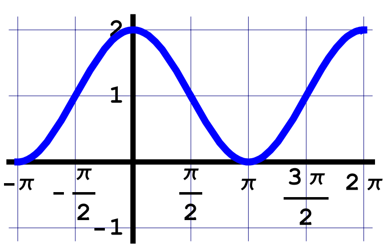
6. $\int \tan^2 3x\,dx$

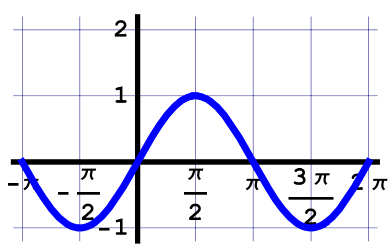
$= \int (\sec^2 3x - 1)\,dx$

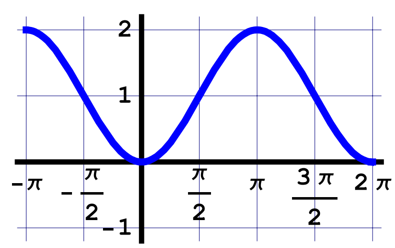
$= \boxed{\frac{1}{3}\tan 3x - x + C}$

7. Determine which of the following graphs is a solution to the differential equation

$\frac{dy}{dx} = \sin x,\ y\left(\frac{\pi}{2}\right) = 1 \quad \rightarrow \quad y = -\cos x + C \quad 0 + C = 1 \quad C = 1 \quad \boxed{y = -\cos x + 1}$







Solve the initial value problem.

8. $f'(x) = 12x^2 - 6x + 1,\ f(1) = 5$

$f(x) = 4x^3 - 3x^2 + x + C$ so

$4 - 3 + 1 + C = 5 \rightarrow C = 3$ so

$\boxed{f(x) = 4x^3 - 3x^2 + x + 3}$

9. $\frac{d^2y}{dx^2} = 2\cos x - 5\sin x,\ y(\pi) = 2 + 6\pi$, and $y'(\pi) = 3$

$\frac{dy}{dx} = 2\sin x + 5\cos x + C \rightarrow 0 - 5 + C = 3 \rightarrow C = 8$

$\frac{dy}{dx} = 2\sin x + 5\cos x + 8 \rightarrow y = -2\cos x + 5\sin x + 8x + D$

$\rightarrow 2 + 8\pi + D = 2 + 6\pi \quad D = -2\pi$ and

$\boxed{y = -2\cos x + 5\sin x + 8x - 2\pi}$

10. $a(t) = 2 - 6t, \quad v(0) = -5, \quad s(0) = 4$

$v(t) = 2t - 3t^2 + C \qquad C = -5 \qquad$ so

$v(t) = 2t - 3t^2 - 5 \qquad$ and

$s(t) = t^2 - t^3 - 5t + D \qquad D = 4 \qquad$ so

$$\boxed{s(t) = t^2 - t^3 - 5t + 4}$$

11. Verify the formula $\displaystyle\int \frac{-1}{|x|\sqrt{x^2-1}}\,dx = \csc^{-1}x + C$

$$D_x \int \frac{-1}{|x|\sqrt{x^2-1}}\,dx = D_x\left(\csc^{-1}x + C\right)$$

$$\boxed{\frac{-1}{|x|\sqrt{x^2-1}} = \frac{-1}{|x|\sqrt{x^2-1}}}$$

12. A right circular cylindrical tank with radius 8 feet and height 12 feet that was initially full of water is being drained at the rate of $2\sqrt{h}\ \dfrac{ft^3}{min}$ Find a formula for the depth and the amount of water in the tank at any time t

$V = \pi r^2 h = 64\pi h \rightarrow \dfrac{dV}{dt} = 64\pi\dfrac{dh}{dt} = -2\sqrt{h} \rightarrow h^{\frac{-1}{2}}\dfrac{dh}{dt} = \dfrac{-1}{32\pi}$ so

$\displaystyle\int h^{\frac{-1}{2}}\frac{dh}{dt}\,dt = \int \frac{-1}{32\pi}\,dt \rightarrow \int h^{\frac{-1}{2}}\,dh = \frac{-1}{32\pi}t + C \rightarrow 2\sqrt{h} = \frac{-1}{32\pi}t + C$ and

when $t = 0, \ h = 12,$ so $\quad 2\sqrt{12} = C \rightarrow 2\sqrt{h} = \dfrac{-1}{32\pi}t + 4\sqrt{3}$ and

$\boxed{h = \left(\dfrac{-1}{64\pi}t + 2\sqrt{3}\right)^2}$ so $\boxed{V = 64\pi\left(\dfrac{-1}{64\pi}t + 2\sqrt{3}\right)^2}$

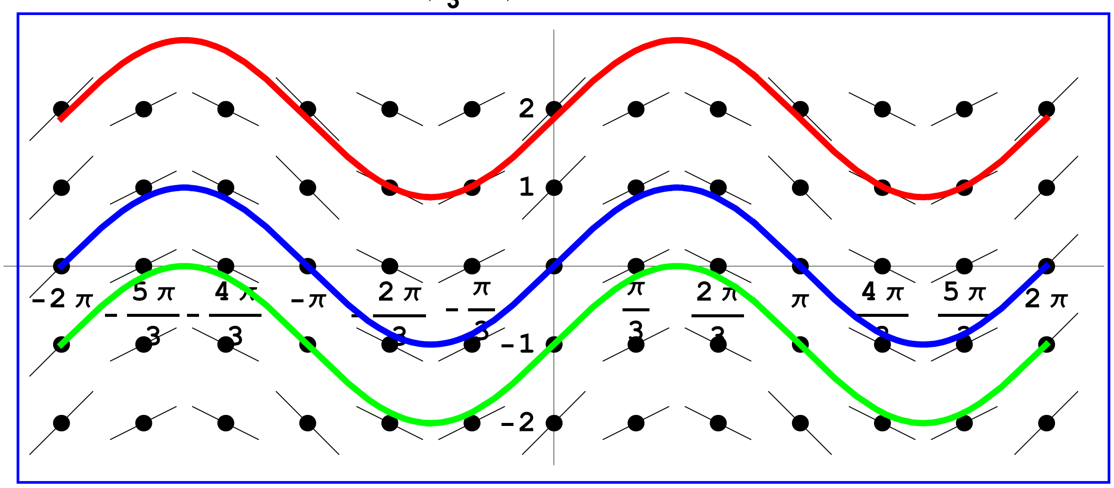
13. Suppose \$800 is invested in an account that pays 5.25% interest compounded continuously.
(a) Find a formula for the amount in the account at any time t.
(b) When will the amount in the account be four times the initial amount?

(a) $P = P_0 e^{rt}$ so $\boxed{P = 800e^{0.0525t}}$ (b) $3200 = 800e^{0.0525t} \rightarrow 4 = e^{0.0525t}$ so

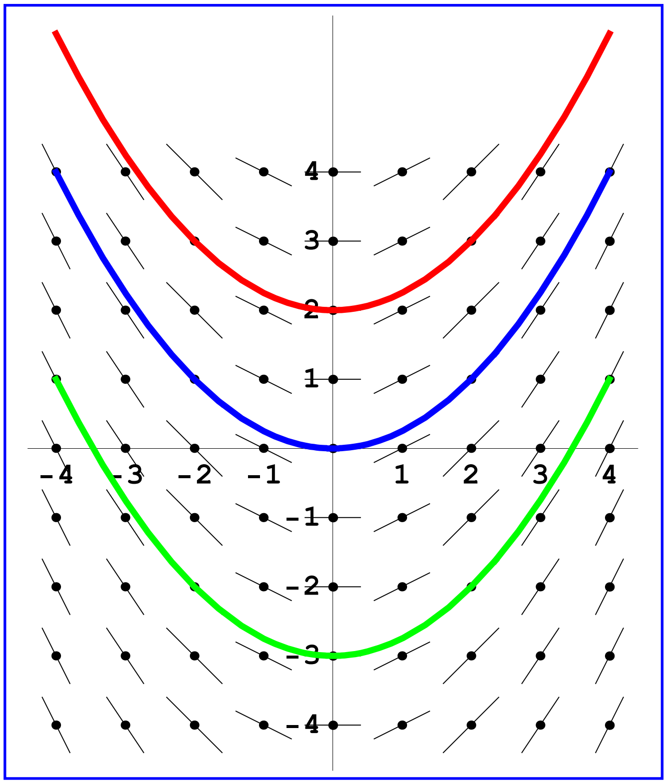
$\ln 4 = 0.0525t$ and $\boxed{t = \dfrac{\ln 4}{0.0525} \text{ years}}$

Using the generated slope field for each differential equation, draw a graphical solution through each of the given points.

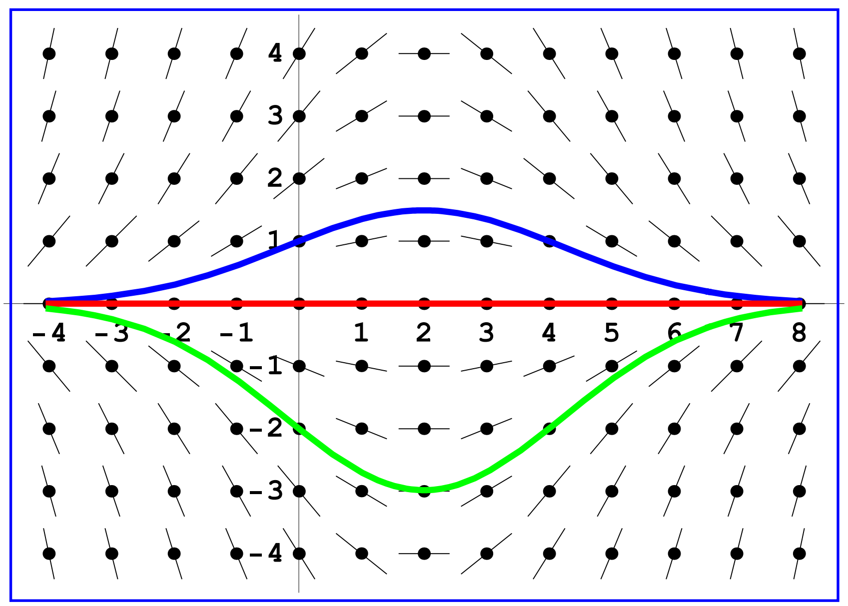
14. $y' = \cos x$ (a) $(0, 0)$ (b) $\left(\dfrac{-\pi}{3}, 1\right)$ (c) $(\pi, -1)$



15. $\dfrac{dy}{dx} = \dfrac{x}{2}$ (a) $(0, 0)$ (b) $(2, 3)$ (c) $(-2, -2)$



16. $y' = \dfrac{(2-x)y}{5}$ **(a)** (0, 1) **(b)** (3, 0) **(c)** (4, −2)



17. $\dfrac{dy}{dx} = \dfrac{x}{y}$ (a) (1, 1) (b) (0, 3) (c) (4, 2)