

6.1 Antiderivatives and Slope Fields

Integral Formulas

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

$$\int \frac{1}{x} dx = \ln|x| + C$$

$$\int e^{kx} dx = \frac{1}{k} e^{kx} + C$$

$$\int \sin kx dx = -\frac{1}{k} \cos kx + C$$

$$\int \cos kx dx = \frac{1}{k} \sin kx + C$$

$$\int \sec^2 x dx = \tan x + C$$

$$\int \csc^2 x dx = -\cot x + C$$

$$\int \sec x \tan x dx = \sec x + C$$

$$\int \csc x \cot x dx = -\csc x + C$$

$$\int f(x) dx = F(x) + C \quad \text{where } F'(x) = f(x)$$

$$\int kf(x) dx = k \int f(x) dx$$

$$\int (f(x) \pm g(x)) dx = \int f(x) dx \pm \int g(x) dx$$

Evaluate the integral.

1. $\int (2x^3 - x^2 + 3x - 7) dx$

2. $\int \left(\sqrt{u^3} - \frac{1}{2}u^{-2} + \sqrt[3]{u} \right) du$

3. $\int \frac{8x - 5}{\sqrt[4]{x}} dx$

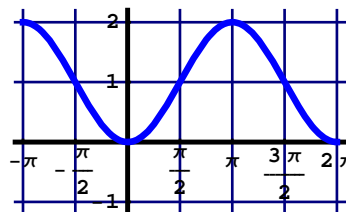
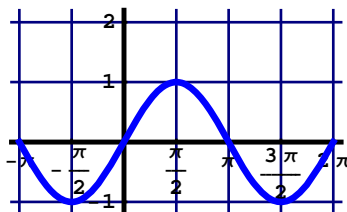
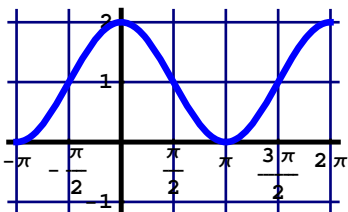
4. $\int \left(\cos 3x - \frac{1}{x+4} + e^{-2x} \right) dx$

5. $\int (-3 \csc 2t \cot 2t) dt$

6. $\int \tan^2 3x dx$

7. Determine which of the following graphs is a solution to the differential equation

$$\frac{dy}{dx} = \sin x, \quad y\left(\frac{\pi}{2}\right) = 1$$



Solve the initial value problem.

8. $f'(x) = 12x^2 - 6x + 1$, $f(1) = 5$

9. $\frac{d^2y}{dx^2} = 2 \cos x - 5 \sin x$, $y(\pi) = 2 + 6\pi$, and $y'(\pi) = 3$

10. $a(t) = 2 - 6t$, $v(0) = -5$, $s(0) = 4$

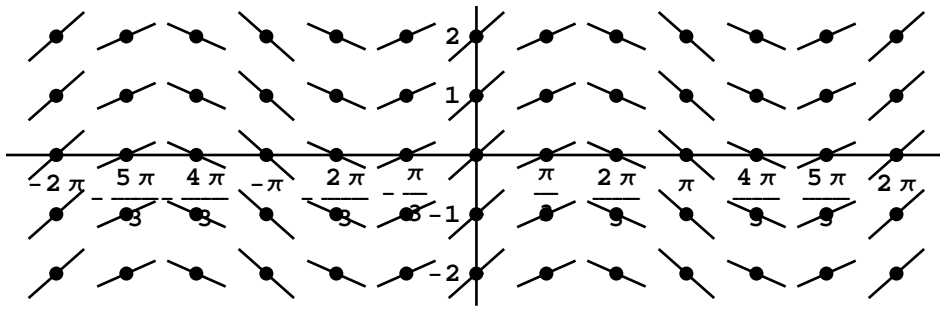
11. Verify the formula $\int \frac{-1}{|x| \sqrt{x^2 - 1}} dx = \csc^{-1} x + C$

12. A right circular cylindrical tank with radius 8 feet and height 12 feet that was initially full of water is being drained at the rate of $2\sqrt{h} \frac{\text{ft}^3}{\text{min}}$. Find a formula for the depth and the amount of water in the tank at any time t .

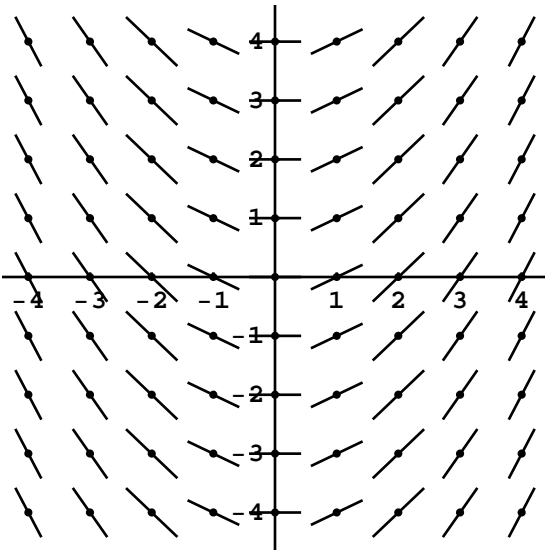
13. Suppose \$800 is invested in an account that pays 5.25% interest compounded continuously.
(a) Find a formula for the amount in the account at any time t .
(b) When will the amount in the account be four times the initial amount?

Using the generated slope field for each differential equation, draw a graphical solution through each of the given points.

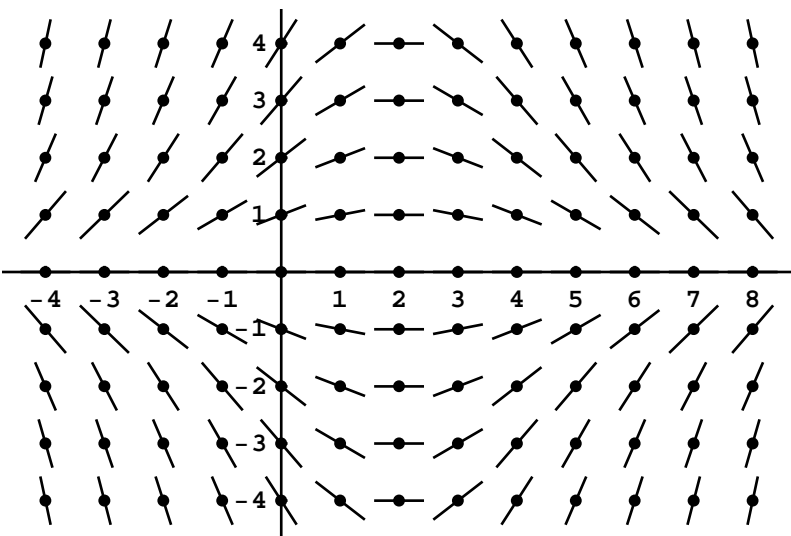
14. $y' = \cos x$ (a) $(0, 0)$ (b) $\left(\frac{-\pi}{3}, 1\right)$ (c) $(\pi, -1)$



15. $\frac{dy}{dx} = \frac{x}{2}$ (a) $(0, 0)$ (b) $(2, 3)$ (c) $(-2, -2)$



16. $y' = \frac{(2-x)y}{5}$ (a) $(0, 1)$ (b) $(3, 0)$ (c) $(4, -2)$

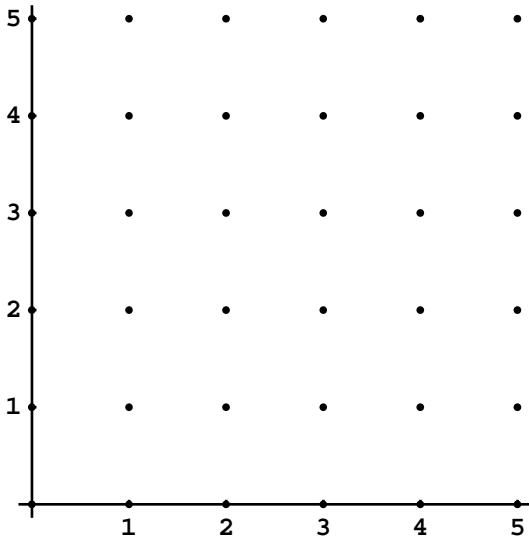


17. $\frac{dy}{dx} = \frac{x}{y}$

(a) (1, 1)

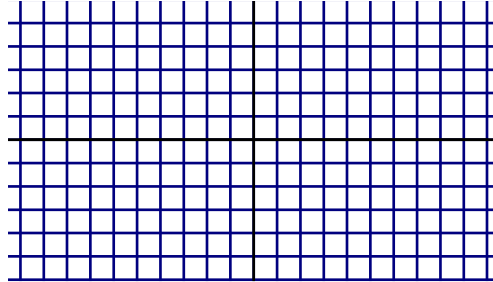
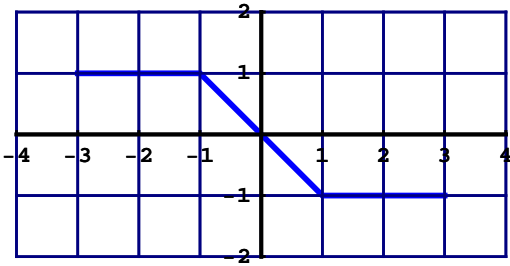
(b) (0, 3)

(c) (4, 2)

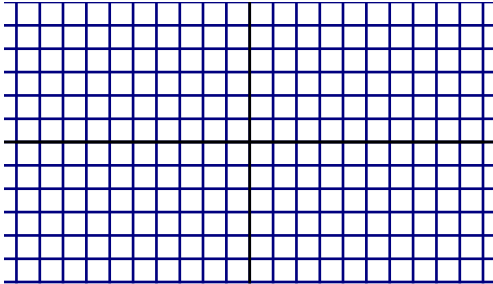
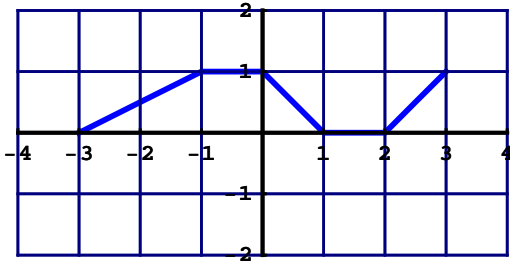


For problems 18 – 20, given the graph of $f'(x)$, sketch the graph of $f(x)$.

18.



19.



20.

