6.2 Integration by Substitution

$$\int f(g(x))\, g'(x)\, dx = \int f(u)\, du \qquad \text{where } u = g(x) \text{ and } du = g'(x)\, dx$$

$$\int_a^b f(g(x))\, g'(x)\, dx = \int_{g(a)}^{g(b)} f(u)\, du \qquad \text{where } u = g(x) \text{ and } du = g'(x)\, dx$$

The trick is to use up the whole integrand (and dx) with some combination of a function and its derivative. The du (or the derivative) must exist as a multiple. The u (or the function) can be nested as part of a composition, the denominator, or . . .

Evaluate the integral.

1. $\displaystyle \int \frac{5x}{\sqrt{x^2 + 3}}\, dx$

2. $\displaystyle \int \frac{\left(1 + \sqrt{x}\right)^3}{\sqrt{x}}\, dx$

3. $\displaystyle \int \frac{x^2 - 1}{(x^3 - 3x + 1)^6}\, dx$

4. $\displaystyle \int \frac{3}{\sqrt{4 - 5t}}\, dt$

5. $\displaystyle \int \cos^3 5x \ \sin 5x \ dx$

6. $\displaystyle \int (s^2 + 1)^2\, ds$

7. $\displaystyle \int \frac{1}{x(\log_4 x)^2}\, dx$

8. $\displaystyle \int \frac{1}{\sqrt{9 - x^2}}\, dx \qquad u = \frac{x}{3}$

9. $\displaystyle \int \frac{\sin 4x}{\cos 2x}\, dx$

10. $\displaystyle\int \frac{\sin y}{1 + \cos^2 y}\, dy$

11. $\displaystyle\int_{\frac{-\pi}{4}}^{\frac{\pi}{3}} \tan x\, dx$

12. $\displaystyle\int_{\frac{\pi}{2}}^{\frac{5\pi}{6}} \sqrt{\csc^5 \theta}\, \cot\theta\, d\theta$

13. $\displaystyle\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} (x + \sin 5x)\, dx$

14. $\displaystyle\int_{0}^{1} \frac{1}{(3 - 2x)^2}\, dx$

15. $\displaystyle\int_{-1}^{2} 4^x \sqrt{1 + 4^x}\, dx$

16. $\displaystyle\int_{0}^{\frac{1}{2}} \frac{\sin^{-1} x}{\sqrt{1 - x^2}}\, dx$

17. $\displaystyle\int_{1}^{3} \frac{e^x + 1}{e^x}\, dx$

18. $\displaystyle\int_{0}^{4} x\sqrt{16 - 3x}\, dx$

19. $\displaystyle\int_{-1}^{1} \frac{x + x^3 + x^5}{1 + x^2 + x^4}\, dx$