6.3 Integration by Parts (the Product Rule "backwards")

$\int u\,dv = uv - \int v\,du$

If $u = f(x)$ and $v = g(x)$, then $D_x(uv) = u'v + uv'$ or $D_x(f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$

Now, if we integrate both sides, we get: $\int (D_x(f(x)g(x)))\,dx = \int (f'(x)g(x))\,dx + \int (f(x)g'(x))\,dx$

or $f(x)g(x) = \int g(x)(f'(x)\,dx) + \int f(x)(g'(x)\,dx)$ and, going back to u and v, we have

$uv = \int v\,du + \int u\,dv$ or $\int u\,dv = uv - \int v\,du$

Use u = L ogarithmic
I nverse Trigonometric
P olynomial
E xponential
T rigonometric

The "trick" for using this technique correctly is to choose the dv properly – dv should be the derivative of something times dx. The dv should also take up as much as possible of the integrand.

1. $\int x \tan^{-1} x\,dx$

2. $\int \sqrt{x}\,\log_2 x\,dx$

3. $\int (x + 5)\csc^2 4x\,dx$

4. $\int x^3 \cos 2x\,dx$

5. $\int x^4 e^{-3x}\,dx$

6. $\int e^{3x} \cos 2x\,dx$

7. $\int \cot^{-1} 3x \; dx$

8. $\int \cos(\ln x)\, dx$

9. $\int x^5 e^{-x^2} \; dx$

10. $\int_0^{\frac{\pi}{2}} x \sin 2x \; dx$

11. $\int_0^{\frac{\pi}{4}} \sec^3 x \; dx$