

6.4 Exponential Growth and Decay

Growth and Decay Equations

$$\frac{dy}{dt} = ky \quad \text{and} \quad y = y_0 e^{kt}$$

Money compounding k times a year, where r is the rate

$$A(t) = A_0 \left(1 + \frac{r}{k}\right)^{kt}$$

Money compounding continuously

$$A(t) = A_0 e^{rt}$$

Newton's Law of Cooling, $\frac{dT}{dt} = -k(T - T_s)$ and $T - T_s = (T_0 - T_s)e^{-kt}$

where T_0 is the initial temperature, and T_s is the surrounding temperature

Motion of a Coasting Object, $\frac{dv}{dt} = \frac{-k}{m}v$, and $v = v_0 e^{-(\frac{k}{m})t}$ where $k > 0$, v_0 is the initial velocity, and m is the mass.

Solve the following differential equations in 1 and 2.

1. $\frac{dy}{dx} = x^2 e^{-y} \sec e^y$

2. $\frac{dy}{dx} = \frac{(1 + y^2)^2}{xy \ln x}$

Find the solution to $\frac{dy}{dt} = ky$, with the given initial conditions in 3 and 4.

3. $k = 0.5$, $y(0) = 200$

4. $y(0) = 80$, $y(20) = 400$

For an investment compounding continuously, find the missing values in 5 and 6.

5. If the initial deposit is \$300, and it takes 10 years for the investment to double in value, how much money will be in the account after 30 years?
6. If the annual rate is 6.5%, and the amount of money after 20 years will be \$20,000, find the initial amount in the account, and how long it will take for the investment to double.

7. Radium has a half – life of approximately 1600 years. Find a formula for the amount of radium remaining if the initial amount is 50 milligrams. Find when there will be 40 milligrams left.
8. A cup of coffee is placed in the microwave and brought to a temperature of 125° . The cup is removed from the microwave and placed on the counter, where the room temperature is 75° . Find a formula for the temperature of the cup if it cools to 100° in half an hour. Find the temperature at the end of another half an hour.

9. Veterinarians use sodium pentobarbital to anesthetize animals. Suppose that to anesthetize a dog, 30 milligrams are required for each kilogram of body weight. If the drug is eliminated exponentially from the bloodstream, and half is eliminated in 4 hours, approximate the single dose that will anesthetize a 20 kg dog for 45 minutes.