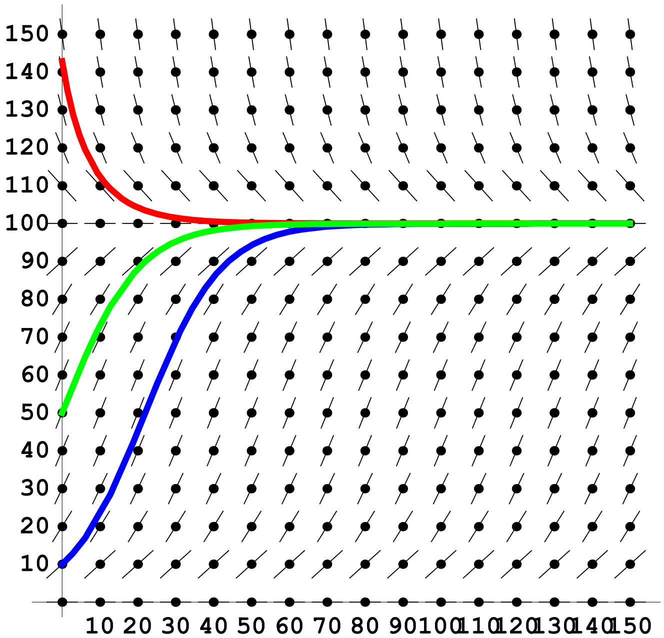
6.5 Population Growth

Growth and Decay Equations $\frac{dy}{dt} = ky$ and $y = y_0 e^{kt}$ where k is the relative growth rate.

Logistic Growth Model $\frac{dP}{dt} = \frac{k}{M} P(M - P)$ and $P = \frac{M}{1 + Ae^{-kt}}$ $\rightarrow$ $A = \frac{M - P_0}{P_0}$

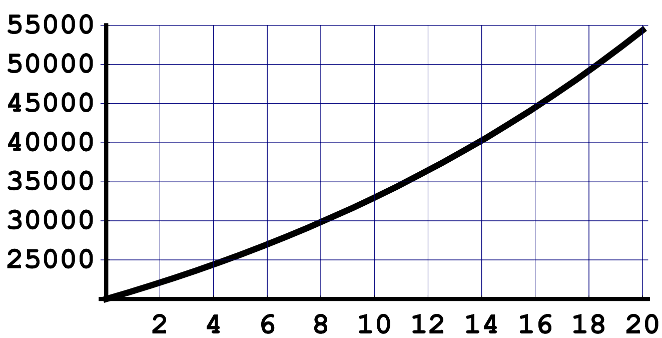
where M is the carrying capacity, with constant k, and initial population P_0.

Here's an example of what a Logistic growth model might look like:



1. Find an equation for the population of Reedley, if the relative growth rate is 0.05, and the current population is 20, 000.

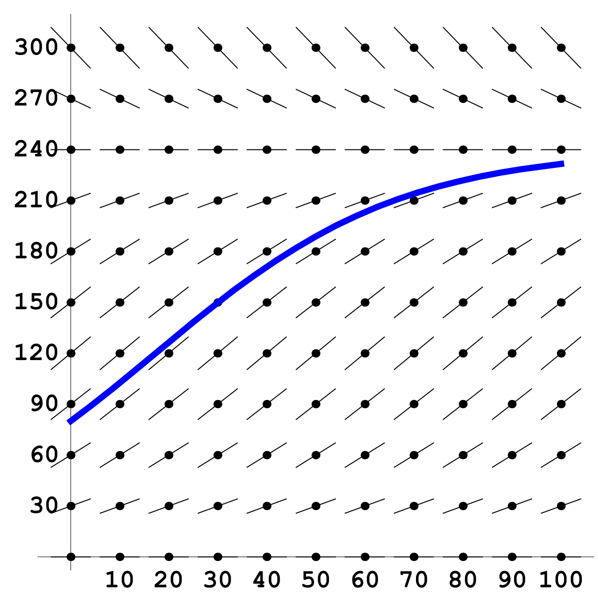
$y = y_0 e^{kt}$ $\boxed{y = 20000 e^{0.05t}}$



2. Find a logistic growth model if $k = 0.04$, $M = 240$, and $P(0) = 80$

$\frac{dP}{dt} = \frac{k}{M} P(M - P)$ and

$P = \frac{M}{1 + \frac{M - P_0}{P_0} e^{-kt}} = \boxed{\frac{240}{1 + 2e^{-0.04t}}}$



Find solutions to the following logistic differential equations in 3 – 5

3. $\dfrac{dP}{dt} = 0.15P\left(1 - \dfrac{P}{120}\right)$

$\dfrac{dP}{dt} = \dfrac{k}{M} P(M - P)$

$\dfrac{0.15}{120} P(120 - P)$

$k = 0.15, \quad M = 120$

$$\boxed{P = \dfrac{120}{1 + \frac{120 - P_0}{P_0} e^{-0.15t}}}$$

4. $\dfrac{dP}{dt} = 0.03P - 0.00006P^2$

$\dfrac{dP}{dt} = \dfrac{k}{M} P(M - P)$

$= 0.03P\left(1 - \dfrac{2}{1000} P\right)$

$= 0.03P\left(1 - \dfrac{P}{500}\right)$

$\dfrac{dP}{dt} = \dfrac{0.03}{500} P(500 - P)$

$k = 0.03, \quad M = 500$

$$\boxed{P = \dfrac{500}{1 + \frac{500 - P_0}{P_0} e^{-0.03t}}}$$

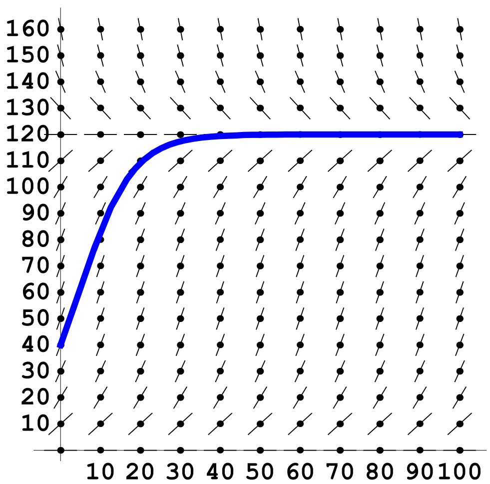
5. $\dfrac{60}{P} \dfrac{dP}{dt} = 4 - \dfrac{P}{200}$

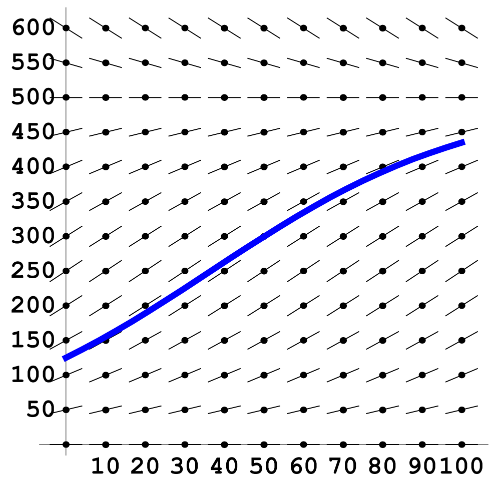
$\dfrac{dP}{dt} = \dfrac{1}{15} P - \dfrac{P^2}{60(200)}$

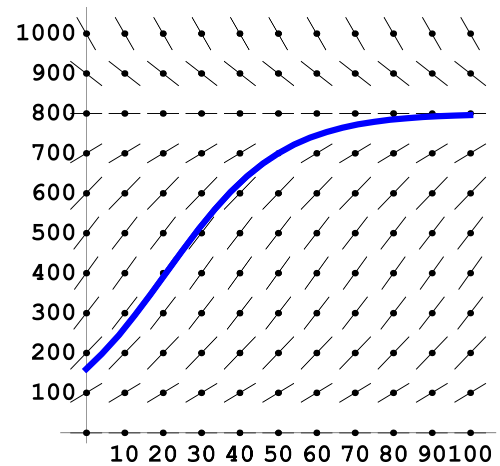
$= \dfrac{1}{15} P \left(1 - \dfrac{P}{800}\right)$

$M = 800, \quad k = \dfrac{1}{15}$

$$\boxed{P = \dfrac{800}{1 + \frac{800 - P_0}{P_0} e^{\frac{-1}{15} t}}}$$







Identify k, the carrying capacity, and P_0 for the following logistic differential equations in 6 and 7.

6. $P(t) = \dfrac{550}{1 + e^{2.5 - 0.5t}}$

$= \dfrac{550}{1 + e^{2.5} e^{-0.5t}}$

$\boxed{M = 550, \quad k = \dfrac{1}{2}}, \quad P_0 = ?$

$e^{2.5} = \dfrac{550 - P_0}{P_0}$

$P_0 e^{2.5} + P_0 = 550 \qquad \boxed{P_0 = \dfrac{550}{1 + e^{2.5}}}$

7. $P(t) = \dfrac{800}{1 + e^{-t}}$

$\boxed{M = 800, \quad k = 1}$

$1 = \dfrac{800 - P_0}{P_0}$ so $\boxed{P_0 = 400}$