6.6 Euler's Method

Euler's Method is used to approximate the solution to a differential equation by using Δx increments

$y_{n+1} = y_n + f(x_n, y_n)\Delta x$, where $\dfrac{dy}{dx} = f(x, y)$

Show that the given function is a solution of the differential equation.

1. $y' = \dfrac{-2y}{3x}$ $y = -3x^{\frac{-2}{3}}$ $y' = 2x^{\frac{-5}{3}}$ Now, substitute in the original differential equation

$y' = \dfrac{-2y}{3x}$ becomes $\dfrac{2}{x^{\frac{5}{3}}} = \dfrac{6x^{\frac{-2}{3}}}{3x} = \dfrac{2}{x^{\frac{5}{3}}}$ Hey, it works!

Solve the following separable differential equations in 2 and 3.

2. $\dfrac{dy}{dx} = x - 1 + xy - y$

$= (x-1) + y(x-1) = (y+1)(x-1)$ and

$\displaystyle\int \frac{dy}{y+1} = \int (x-1)\,dx$ becomes

$\ln|y+1| = \dfrac{1}{2}x^2 - x + C$

3. $\dfrac{dy}{dx}e^{2x} = \csc y$ and $y = 0$ when $x = 0$

$\displaystyle\int \sin y\,dy = \int e^{-2x}\,dx$

$-\cos y = \dfrac{-1}{2}e^{-2x} + C$

$y = \cos^{-1}\left(\dfrac{1}{2}e^{-2x} + C_1\right)$ $\dfrac{1}{2} + C_1 = 1$

$y = \cos^{-1}\left(\dfrac{1}{2}e^{-2x} + \dfrac{1}{2}\right)$

Use the first 3 steps for Euler's Method in problems 4 – 7

4. $\dfrac{dy}{dx} = \dfrac{y}{x}$ $y(2) = 2$ $\Delta x = 0.1$

$y_0 = 2$ $y_1 = 2 + \dfrac{2}{2}(0.1) = 2.1$

$y_2 = 2.1 + \dfrac{2.1}{2.1}(0.1) = 2.2$

$y_3 = 2.2 + \dfrac{2.2}{2.2}(0.1) = 2.3$

Now, let's compare this to the actual solution

$\dfrac{dy}{dx} = \dfrac{y}{x}$ $\displaystyle\int \frac{1}{y}\,dy = \int \frac{1}{x}\,dx$

$\ln|y| = \ln|x| + C$ $y = e^c x$, or,

in this case, $y = x$ Hey, it matches!

5. $y' = \dfrac{y(1 + x^2)}{x^2}$ $y(1) = 1$ $\Delta x = 0.1$

$y_0 = 1$ $y_1 = 1 + 2(0.1) = 1.2$

$y_2 = 1.2 + \dfrac{1.2(1 + 1.1^2)}{1.1^2}(0.1) \approx 1.419$

$y_3 \approx 1.419 + \dfrac{1.419(1 + 1.2^2)}{1.2^2}(0.1) \approx 1.659$

6. $y' = \dfrac{\sqrt{1 - y^2}}{x}$ $y(1) = \dfrac{1}{2}$ $dx = 0.1$

$y_0 = \dfrac{1}{2}$ $y_1 = \dfrac{1}{2} + \dfrac{\sqrt{1 - \frac{1}{4}}}{1}(0.1) \approx 0.587$

$y_2 \approx 0.587 + \dfrac{\sqrt{1 - 0.587^2}}{1.1}(0.1) \approx 0.661$

$y_3 \approx 0.661 + \dfrac{\sqrt{1 - 0.661^2}}{1.2}(0.1) \approx 0.724$

7. $\dfrac{dy}{dx} = x - 1 + xy - y$ $y(1) = -1$ $\Delta x = 0.1$

$y_0 = -1$ $y_1 = -1 + (1 - 1 - 1 + 1)(0.1) = -1$

$y_2 = -1 + (1.1 - 1 - 1.1 + 1)(0.1) = -1$

$y_3 = -1 + (1.2 - 1 - 1.2 + 1)(0.1) = -1$