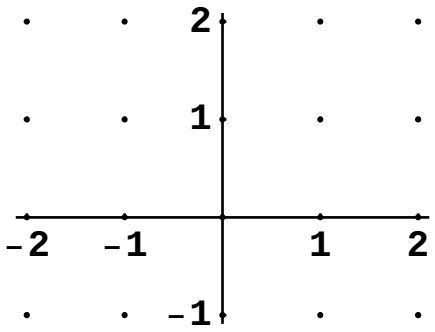
Chapter 6 Test December 5, 2011 Name

1. Evaluate $\int \cot^2 x \sec x \, dx$

2. Evaluate $\int_{-4}^{0} \frac{1}{x^2 + 10x + 25} \, dx$

3. Solve the initial value problem. Support your answer by overlaying your solution on a slope field for the differential equation. $\frac{dy}{dx} = -x + 1$ and $y(2) = 1$



4. Evaluate $\int 3x(3x + 1)^{20} \, dx$

5. Solve the following differential equation by the technique of separation of variables,

$$\frac{1}{3^x e^x}\frac{dy}{dx} = y \quad \text{and} \quad y(0) = 1$$

6. Evaluate $\int \sin\left(2\sqrt{x}\right) dx$

7. Evaluate $\int \left(\cot(3x) + \left(\frac{1}{e^{4x}}\right) - 5x^{\frac{-4}{5}} \right) dx$

8. Evaluate $\int_1^{-1} \frac{4x^3 + 3x}{\sqrt{x^4 + 3x^2 + 5}} dx$

9. Evaluate $\int \frac{\sin 2x}{\sqrt{\frac{1}{2} - \sin^2 x}}\, dx$

10. Evaluate $\int e^{3x} \cos(x)\, dx$

11. Evaluate $\int_1^e \left(\log_3\left(x^2\right)\right)\, dx$

12. A rumor has started that, due to a lack of funding, the current athletic fields project has been suspended. Furthermore, to make the best of the situation, mud pits will be created for P.E. class use. The spread of this rumor is modeled by the equation

$\frac{dP}{dt} = 20P\left(\frac{1}{100} - \frac{P}{50,000}\right)$, with $P = 2$ at $t = 0$ (t in days) representing the number of students (or teachers!) starting the rumor. If $\frac{dP}{dt} = \frac{k}{M}P(M - P)$ and $P = \frac{M}{1 + Ae^{-kt}}$, $A = \frac{M - P_0}{P_0}$,

(a) Find a solution to this differential equation.

(b) Find the number of students who have heard the rumor after 60 days.

13. The growth of Spotify (the online music service) is directly proportional to the number of active users that participate in the service. Currently, there are 10 million active users, and the site is growing (at the moment) at the rate of 4000 users per day. Given this growth rate, how long will it take for Spotify to grow to 25 million active users?

14. Use Euler's Method to numerically solve the initial value problem $y' = xy + 2y$, $y(0) = -1$, on the interval $0 \le x \le 3$, starting at $x_0 = 0$ and $y_0 = -1$ with $\Delta x = 1$.