7.1 The Integral as Net Change

Displacement is the change in position (could be positive, negative, or zero).

Total Distance is always non – negative → think of adding up all of the distances (right or left) traveled.

For problems 1 – 4, determine
(a) when the particle is moving to the right, to the left, and stopped
(b) the particle's displacement for the given time interval
(c) the total distance travelled by the particle

1. $v(t) = 4\cos 2t \qquad 0 \le t \le \frac{2\pi}{3}$

(a) Find when $v(t)$ is zero first, so

$$4\cos 2t = 0 \quad \rightarrow \quad 2t = \frac{\pi}{2} \quad \text{so} \quad t = \frac{\pi}{4}$$

Particle is stopped when $t = \frac{\pi}{4}$

particle is moving to the right on $\left[0, \frac{\pi}{4}\right)$

particle is moving to the left on $\left(\frac{\pi}{4}, \frac{2\pi}{3}\right]$

(b) $\int_0^{\frac{2\pi}{3}} 4\cos 2t \, dt = \left[2\sin 2t \right]_0^{\frac{2\pi}{3}}$

$$= 2\left(\frac{-\sqrt{3}}{2}\right) = \boxed{-\sqrt{3}}$$

(c) $\int_0^{\frac{2\pi}{3}} \left| 4\cos 2t \right| dt =$

$$\int_0^{\frac{\pi}{4}} 4\cos 2t \, dt - \int_{\frac{\pi}{4}}^{\frac{2\pi}{3}} 4\cos 2t \, dt$$

$$= \left[2\sin 2t \right]_0^{\frac{\pi}{4}} - \left[2\sin 2t \right]_{\frac{\pi}{4}}^{\frac{2\pi}{3}}$$

$$= 2(1 - 0) - 2\left(\frac{-\sqrt{3}}{2} - 1\right) = \boxed{4 + \sqrt{3}}$$

2. $v(t) = \sqrt{t + 4} \qquad 0 \le t \le 5$

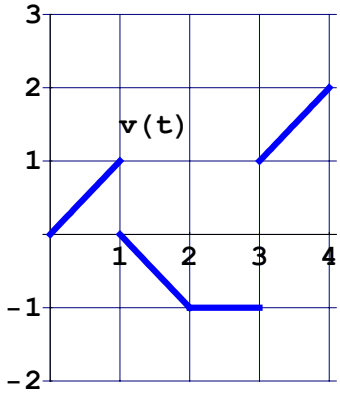
(a) The particle is moving to the right on [0, 5], and is never stopped and never moves to the left

(b) $\int_0^5 \sqrt{t + 4} \, dt = \left[\frac{2}{3}(t + 4)^{\frac{3}{2}} \right]_0^5$

$$= \frac{2}{3}(27 - 8) = \frac{38}{3} = \boxed{12\frac{2}{3}}$$

(c) The particle is always moving to the right, so we get the same answer that we had in (b), $\boxed{12\frac{2}{3}}$

3.



(a) stopped at $t = 0$,

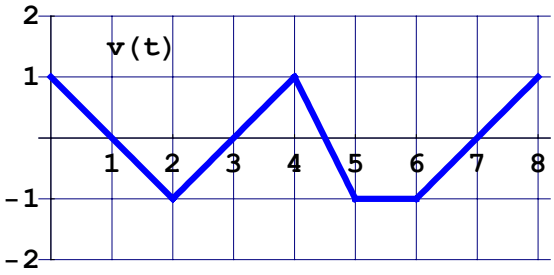
moving to the right on $(0, 1)$ and $(3, 4]$,

and moving to the left $(1, 3)$

(b) displacement is $\frac{1}{2} - \frac{1}{2} - 1 + \frac{3}{2} = \boxed{\frac{1}{2}}$

(c) $\frac{1}{2} + \frac{1}{2} + 1 + \frac{3}{2} = \frac{7}{2} = \boxed{3\frac{1}{2}}$

4.



(a) stopped at $t = 1, 3, 4\frac{1}{2}, 7,$ and

moving to the right on $[0, 1)$, $\left(3, 4\frac{1}{2}\right)$, and $(7, 8]$

and moving to the left on $(1, 3)$, and $\left(4\frac{1}{2}, 7\right)$

(b) displacement is $\frac{1}{2} - \frac{1}{2} - \frac{1}{2} + \frac{1}{2} - \frac{1}{4} + \frac{1}{4}$

$- 1 - \frac{1}{2} + \frac{1}{2} = \boxed{-1}$

(c) $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{4} + \frac{1}{4} + 1 + \frac{1}{2} + \frac{1}{2} = \boxed{4\frac{1}{2}}$