7.1 The Integral as Net Change (2nd day)

Work $= \int_a^b \text{force}\,dx$ or $W = \int_a^b f(x)\,dx$ where W represents the work, and f(x) is the force

Hooke's Law → $F = kx$ → The force, F, required to stretch a spring x units beyond its natural length is k (the force, or spring, constant) times x

1. The rate of consumption of french fries in the United States can be modeled by the function $C(t) = 1.5 + 1.2^t$, measured in millions of pounds per year, beginning at the start of the year 1990. Find how many pounds of french fries were consumed from the start of 1995 to the end of 1998.

$$\int_5^9 (1.5 + 1.2^t)\,dt = \left[1.5t + \frac{1}{\ln 1.2} 1.2^t \right]_5^9 = \boxed{\left(6 + \frac{1}{\ln \frac{6}{5}} (1.2^9 - 1.2^5) \right) \text{ million pounds}}$$

$$\approx \boxed{20.653 \text{ million pounds}}$$

2. Worldwide consumption of Coca – Cola$_{TM}$ continues to grow. Below is a table of the rate of Coke$_{TM}$ consumption, measured in billions of gallons per year. Estimate the number of gallons of Coke$_{TM}$ consumed during the period from the beginning of 1980 to the beginning of 2000 using
   (a) a left – hand rectangle approximation
   (b) the Trapezoidal Rule

Year	Rate (billions of gallons / year)
1980	3.4
1984	3.8
1988	4.5
1992	5.3
1996	5.9
2000	6.8

(a) $4(3.4 + 3.8 + 4.5 + 5.3 + 5.9) = \boxed{91.6 \text{ billions of gallons}}$

(b) $\frac{20 - 0}{2(5)} (3.4 + 2(3.8) + 2(4.5) + 2(5.3) + 2(5.9) + 6.8) = \boxed{98.4 \text{ billions of gallons}}$

3. A certain spring of length 20 cm requires a force of 5 Newtons to stretch it to a length of 24 cm. Find the work done in stretching the spring from a length of 22 cm to a length of 28 cm, and the work done in stretching it from a length of 24 cm to a length of 30 cm.

$f(x) = kx \quad 5 = k(4) \rightarrow k = \frac{5}{4}$ and $f(x) = \frac{5}{4}x$

(a) $W = \int_2^8 \frac{5}{4}x\,dx = \frac{5}{8}\left[x^2 \right]_2^8 = \frac{5}{8}(64 - 4) = \boxed{\frac{75}{2} \text{ Newtons} \times \text{cms}}$

(b) $\int_4^{10} \frac{5}{4}x\,dx = \frac{5}{8}\left[x^2 \right]_4^{10} = \frac{5}{8}(10 - 16) = \boxed{\frac{105}{2} \text{ N} \times \text{cms}}$