7.3 Washers and Shells

$V_{washer} = \pi\left((\text{outer radius})^2 - (\text{inner radius})^2\right)(\text{thickness})$

$V_{shell} = 2\pi(\text{radius})(\text{altitude})(\text{thickness})$

Procedure for setting up a volume integral for a solid of revolution:

(1) Determine the independent variable (x or y).

(2) Determine the axis of rotation (x or y).

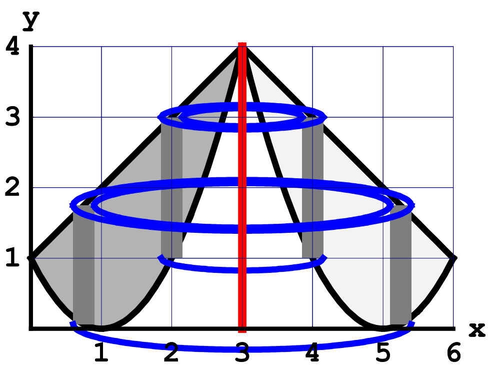
(3) If the variables match, use washers. If the variables don't match, use shells.

(4) Set up all components using top – bottom or right – left.

For problems 1 – 8, find the volume of the indicated solid.

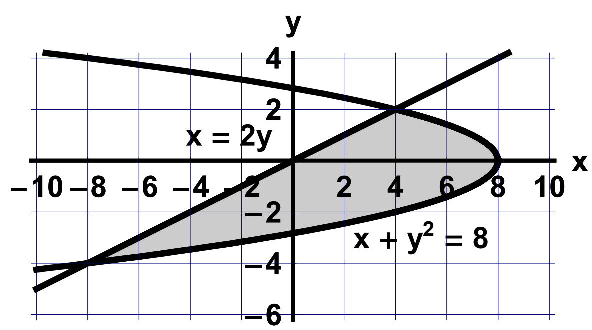
1. $y = x + 1$ and $y = (x-1)^2$, rotated around the line $x = 3$.

$$V_{shells} = 2\pi \int_0^3 (3 - x)\left(x + 1 - (x-1)^2\right) dx$$



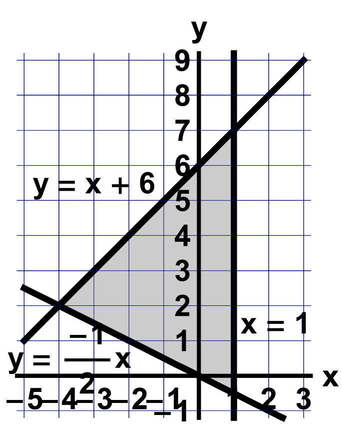
2. $x = 2y$ and $x + y^2 = 8$, rotated around the line $y = 2$.

$$V_{shells} = 2\pi \int_{-4}^{2} (2 - y)(8 - y^2 - 2y)\, dy$$



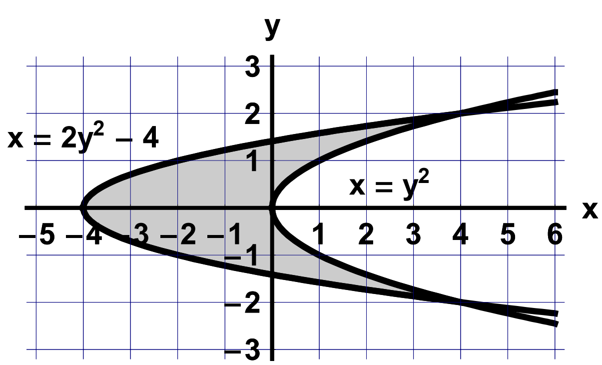
3. $y = x + 6$, $y = \dfrac{-1}{2}x$, and $x = 1$, rotated around the line $y = -1$.

$$V_{washers} = \pi \int_{-4}^{1} \left((x + 6 - (-1))^2 - \left(\frac{-1}{2}x - (-1)\right)^2 \right) dx$$



4. $x = 2y^2 - 4$ and $x = y^2$, rotated around the line $x = 4$.

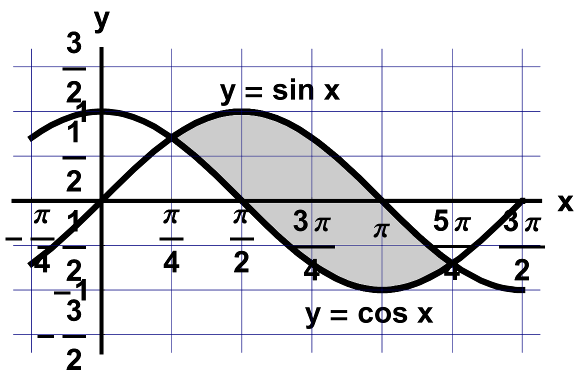
$$V_{washers} = \pi \int_{-2}^{2} \left((4 - (2y^2 - 4))^2 - (4 - y^2)^2 \right) dy$$



5. $y = \sin x$ and $y = \cos x$, on the interval $[\frac{\pi}{4}, \frac{5\pi}{4}]$,

rotated around the line $x = \frac{\pi}{4}$.

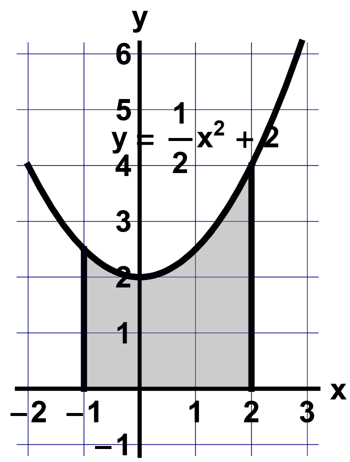
$$V_{shells} = 2\pi \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \left(x - \frac{\pi}{4}\right)(\sin x - \cos x)\, dx$$



6. $y = \frac{1}{2}x^2 + 2$, $y = 0$, $x = -1$, and $x = 2$,

rotated around the line $y = 5$.

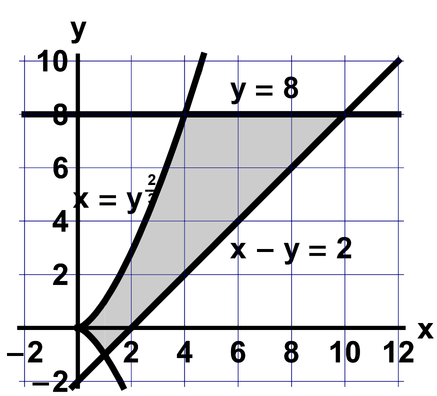
$$V_{washers} = \pi \int_{-1}^{2} \left((5 - 0)^2 - \left(5 - \left(\frac{1}{2}x^2 + 2\right)\right)^2\right) dx$$



7. $x = y^{\frac{2}{3}}$, $y = 8$, and $x - y = 2$,

rotated around the line $y = -2$.

$$V_{shells} = 2\pi \int_{-1}^{8} (y - (-2))\left(y + 2 - y^{\frac{2}{3}}\right) dy$$



8. $y = 2x$ and $y = \frac{1}{8}x^3$,

rotated around the line $y = 8$.

$$V_{washers} = \pi \int_{0}^{4} \left(\left(8 - \frac{1}{8}x^3\right)^2 - (8 - 2x)^2\right) dx$$