7.3 Volume Of A Solid With Known Cross Sections

The volume of a solid of known integrable cross section area $A(x)$ from $x = a$ to $x = b$ can be represented by

$$V = \int_a^b A(x)\,dx$$

Steps for finding the volume of a solid with known cross sections.

(1) Sketch the solid and a typical cross section.

(2) Find a formula for $A(x)$.

(3) Find the limits of integration.

(4) Evaluate the integral $V = \int_a^b A(x)\,dx$

For problems 1 – 6, find the volume of the indicated solid.

1. A solid has as its base the region in the xy – plane bounded by the graphs of $x = 4$ and $x = y^2$. Find the volume of the solid if every cross section by a plane perpendicular to the x – axis is a square with a base in the xy – plane.

2. A solid has as its base the region in the xy – plane bounded by the graph of $x^2 + y^2 = 9$. Find the volume of the solid if every cross section by a plane perpendicular to the x – axis is an equilateral triangle with base in the xy – plane.

3. A solid has as its base the region in the xy – plane bounded by the graph $y = \sin x$ and the x – axis on the interval $[0, \pi]$. Find the volume of the solid if every cross section by a plane perpendicular to the x – axis is a circle with a diameter in the xy – plane.

4. A solid has as its base the region in the xy – plane bounded by the graph of $x^2 + y^2 = 4$. Find the volume of the solid if every cross section by a plane perpendicular to the y – axis is a rectangle with a base in the xy – plane, and height equal to $\frac{1}{2}$ the base.

5. A solid has as its base the region in the xy – plane bounded by the graphs of $y = 9$ and $y = x^2$. Find the volume of the solid if every cross section by a plane perpendicular to the x – axis is a semicircle with a diameter in the xy – plane.

6. A solid has as its base the region in the xy – plane bounded by the graphs of $y = x$ and $y = x^2$. Find the volume of the solid if every cross section by a plane perpendicular to the y – axis is a square with a diagonal in the xy – plane.