7.3 Volume By Shells

$V_{shell} = 2\pi\,(\text{radius})\,(\text{altitude})\,(\text{thickness})$

x – axis $V_{shell} = 2\pi \int_c^d y\,(f(y) - g(y))\,dy$ where y is the radius and

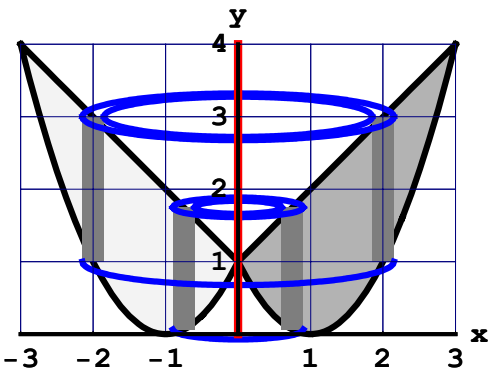
$(f(y) - g(y))$ is the altitude (or height)

y – axis $V_{shell} = 2\pi \int_a^b x\,(f(x) - g(x))\,dx$ where x is the radius and

$(f(x) - g(x))$ is the altitude (or height)

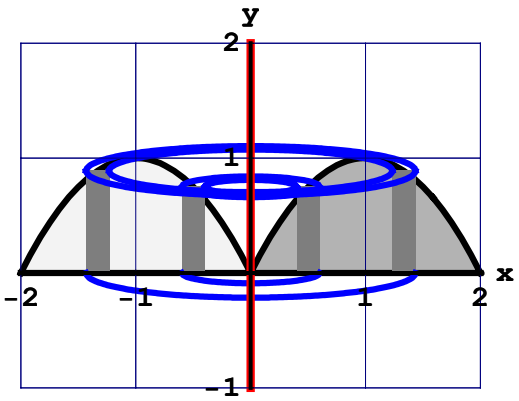
1. $y = x + 1$, $y = (x-1)^2$,
rotated around the y – axis

$$V_{shells} = 2\pi \int_0^3 x\,(x + 1 - (x-1)^2)\,dx$$



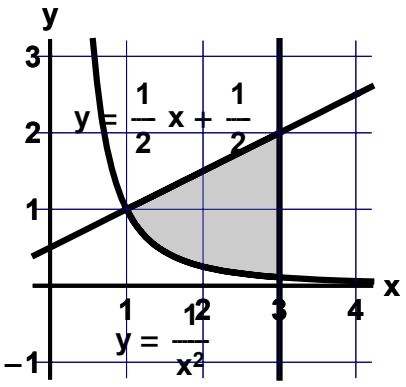
2. $y = 2x - x^2$, $y = 0$,
rotated around the y – axis

$$V_{shells} = 2\pi \int_0^2 x\,(2x - x^2)\,dx$$



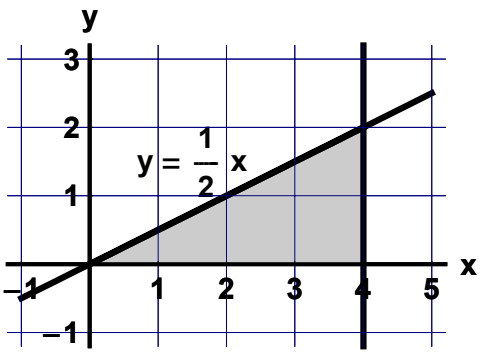
3. $y = \frac{1}{2}x + \frac{1}{2}$, $y = \frac{1}{x^2}$, and $x = 3$,
rotated around the y – axis

$$V_{shells} = 2\pi \int_1^3 x\left(\frac{1}{2}x + \frac{1}{2} - \frac{1}{x^2}\right)dx$$



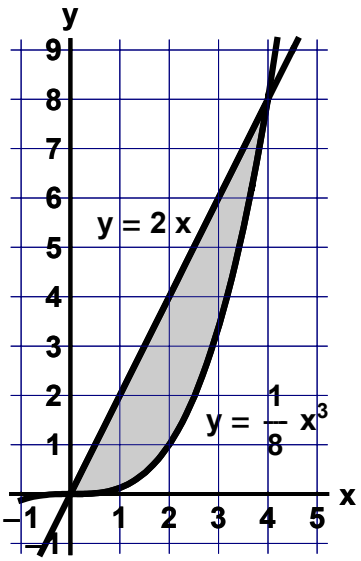
4. $y = \frac{1}{2}x$, $y = 0$, and $x = 4$,
rotated around the x – axis

$$V_{shells} = 2\pi \int_0^2 y\,(4 - 2y)\,dy$$



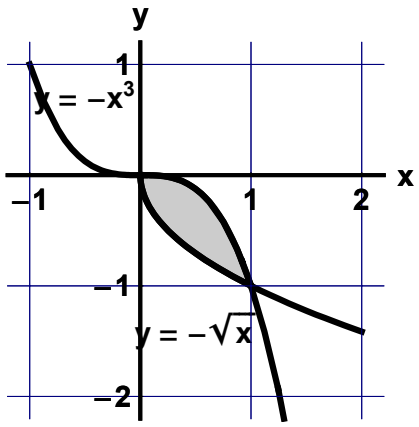
5. $y = 2x$, $y = \frac{1}{8}x^3$, rotated around the y – axis

$$V_{shells} = 2\pi \int_0^4 x\left(2x - \frac{1}{8}x^3\right) dx$$



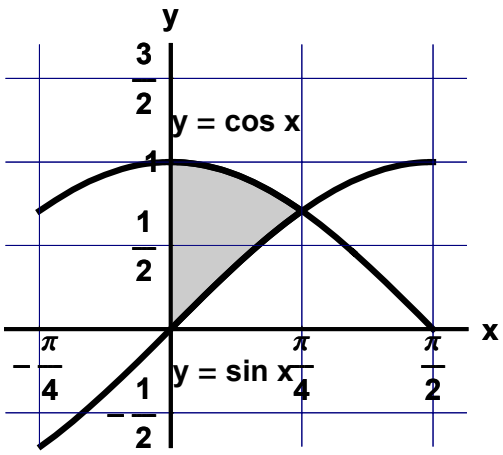
6. $y = -x^3$, and $y = -\sqrt{x}$, rotated around the x – axis

$$V_{shells} = 2\pi \int_{-1}^0 (-y)\left(-y^{\frac{1}{3}} - y^2\right) dy$$



7. $y = \cos x$, $y = \sin x$, and $x = 0$, rotated around the y – axis

$$V_{shells} = 2\pi \int_0^{\frac{\pi}{4}} x(\cos x - \sin x)\, dx$$



8. $x = y^{\frac{2}{3}}$, $x - y = 2$, $y = 2$, and $y = 8$, rotated around the x – axis

$$V = 2\pi \int_2^8 y\left(y + 2 - y^{\frac{2}{3}}\right) dy$$